Lattices for electron storage rings

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Lattice design phases and strategy

Building blocks, magnetic multi-pole expansion

Reminder on matrices and betatron functions

Low emittance lattice conditions

Examples of low emittance lattices
Lattice design phases

- Initial preparation
  - Performance
  - Boundary conditions and constraints
  - Building blocks (magnets)
- Linear lattice design
  - Build modules, and match them together
  - Achieve optics conditions for maximizing performance
  - Global quantities choice working point and chromaticity
- Non-linear lattice design
  - Chromaticity correction (sextupoles)
  - Dynamic aperture
- Real world
  - Include imperfections and foresee corrections
Magnet Design: Technological limits, coil space, field quality
Vacuum: Impedance, pressure, physical apertures, space
Radiofrequency: Energy acceptance, bunch length, space
Diagnostics: Beam position monitors, resolution, space
Alignment: Orbit distortion and correction
Mechanical engineering: Girders, vibrations
Design engineering: Assembly, feasibility

A lattice section....
(top) .....as seen by the lattice designer
(bottom) ..... as seen by the design engineer
(right) ..... and how it looks in reality
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A typical lattice for a storage ring

**SOLEIL 2.75 GeV**
- 354 m long ring
- 24 Straight sections
- 1 Injection system
- 2 RF cavities
- 21 for ID’s

**Focusing magnet**
**QUADRUPOLE**

**Bending magnet**
**DIPOLE**

**EXPERIMENTAL HALL**

**Beamline Front-End**

**INSERTION DEVICE**
From Gauss law of magnetostatics, a vector potential exist
\[ \nabla \cdot \mathbf{B} = 0 \quad \rightarrow \quad \exists \mathbf{A} : \quad \mathbf{B} = \nabla \times \mathbf{A} \]

Assuming a 2D field in \( x \) and \( y \), the vector potential has only one component \( A_s \). The Ampere’s law in vacuum (inside the beam pipe)
\[ \nabla \times \mathbf{B} = 0 \quad \rightarrow \quad \exists V : \quad \mathbf{B} = -\nabla V \]

Using the previous equations, the relations between field components and potentials are
\[ B_x = -\frac{\partial V}{\partial x} = \frac{\partial A_s}{\partial y}, \quad B_y = -\frac{\partial V}{\partial y} = -\frac{\partial A_s}{\partial x} \]
i.e. Riemann conditions of an analytic function

There exist a complex potential of \( z = x + iy \) with a power series expansion convergent in a circle with radius \( |z| = r_c \) (distance from iron yoke)
\[ \mathcal{A}(x + iy) = A_s(x, y) + iV(x, y) = \sum_{n=1}^{\infty} \kappa_n z^n = \sum_{n=1}^{\infty} (\lambda_n + i\mu_n)(x + iy)^n \]
Magnetic multipole expansion II

From the complex potential we can derive the fields

$$B_y + iB_x = -\frac{\partial}{\partial x} (A_\text{s}(x, y) + iV(x, y)) = -\sum_{n=1}^{\infty} n(\lambda_n + i\mu_n)(x + iy)^{n-1}$$

Setting $b_n = -n\lambda_n$, $a_n = n\mu_n$ we have

$$B_y + iB_x = \sum_{n=1}^{\infty} (b_n - ia_n)(x + iy)^{n-1}$$

Define normalized units

$$b'_n = \frac{b_n}{10^{-4}B_0} r_0^{n-1}, \quad a_n = \frac{a_n}{10^{-4}B_0} r_0^{n-1}$$

on a reference radius, $10^{-4}$ of the main field to get

$$B_y + iB_x = 10^{-4}B_0 \sum_{n=1}^{\infty} (b'_n - ia'_n)\left(\frac{x + iy}{r_0}\right)^{n-1}$$

Note: $n' = n-1$ is the US convention
**Magnet definitions**

- **2n-pole:**
  - Dipole
  - Quadrupole
  - Sextupole
  - Octupole ...

  ![Magnet Diagrams]

  $n$: 1, 2, 3, 4 ...

- **Normal:** gap appears at the horizontal plane
- **Skew:** rotate around beam axis by $\pi/2n$ angle
- **Symmetry:** rotating around beam axis by $\pi/n$ angle, the field is reversed (polarity flipped)
## Magnetic field and aperture

<table>
<thead>
<tr>
<th></th>
<th>coil width $\frac{L_{\text{tot}}-L_{\text{eff}}}{2}$ [mm]</th>
<th>poletip field $B_{\text{pt}}$ [T]</th>
<th>aperture R [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending magnets:</td>
<td>65 ... 150</td>
<td>1.5</td>
<td>20 ... 35 ($=g/2$)</td>
</tr>
<tr>
<td>Quadrupoles:</td>
<td>40 ... 70</td>
<td>0.75</td>
<td>30 ... 43</td>
</tr>
<tr>
<td>Sextupoles:</td>
<td>40 ... 80</td>
<td>0.6</td>
<td>30 ... 50</td>
</tr>
</tbody>
</table>

A. Streun, CAS 2003

- Coil width should be taken into account for space considerations
- Apertures as large as necessary, as small as possible depending on acceptance imposed by lattice (a few centimeters for all main magnets)
- Current is scaled as the nth power (multi-pole order) of the radius
- Pole-tip field below 1.8T (normal conducting magnets)
Generalized transfer matrix

\[
\begin{pmatrix}
 c_x & \frac{1}{\sqrt{K}} s_x & 0 & 0 & \frac{h}{K} (1 - c_x) \\
 -\sqrt{K} s_x & c_x & 0 & 0 & \frac{h}{\sqrt{K}} s_x \\
 0 & 0 & c_y & \frac{1}{\sqrt{k}} s_y & 0 \\
 0 & 0 & -\sqrt{k} s_y & c_y & 0 \\
 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[
c_x = \cos(\sqrt{K}L) \quad s_x = \sin(\sqrt{K}L) \quad c_y = \cos(\sqrt{k}L) \quad s_y = \sin(\sqrt{k}L)
\]

With \[\sqrt{K} = \sqrt{\frac{1}{\rho^2} - k}\]

Dipoles: \( k = 0 \)  Quadrupoles: \( \frac{1}{\rho^2} = 0 \)  Drifts: \( \frac{1}{\rho^2} = 0 \), \( k = 0 \)
The linear betatron motion of a particle is described by

\[ u(s) = \sqrt{\epsilon \beta(s)} \cos(\psi(s) + \psi_0) + D(s) \frac{\Delta P}{P} \]

and

\[ u'(s) = \sqrt{\frac{\epsilon}{\beta(s)}} (\sin(\psi(s) + \psi_0) + \alpha(s) \cos(\psi(s) + \psi_0)) + D'(s) \frac{\Delta P}{P} \]

with \( \alpha, \beta, \gamma \) the twiss functions

\[ \alpha(s) = -\frac{\beta(s)'}{2}, \quad \gamma = \frac{1 + \alpha(s)^2}{\beta(s)} \]

\( \psi \) the betatron phase

\[ \psi(s) = \int \frac{ds}{\beta(s)} \]

The beta function defines the envelope (machine aperture)

\[ E(s) = \sqrt{\epsilon \beta(s)} \]

Twiss parameters evolve as

\[
\begin{pmatrix}
\beta \\
\alpha \\
\gamma
\end{pmatrix}
_{s_2}
= 
\begin{pmatrix}
m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\
-m_{11}m_{21} & m_{11}m_{22} + m_{12}m_{21} & -m_{22}m_{12} \\
m_{21}^2 & 2m_{22}m_{21} & m_{22}^2
\end{pmatrix}
_{s_1}
\begin{pmatrix}
\beta \\
\alpha \\
\gamma
\end{pmatrix}
_{s_1}
\]
Lattice section transfer matrix

- Generalized transfer matrix

\[
\mathcal{M}_{0 \rightarrow s} = \begin{pmatrix}
\sqrt{\frac{\beta(s)}{\beta_0}} (\cos \Delta \psi + \alpha_0 \sin \Delta \psi) & \sqrt{\frac{\beta_0}{\beta(s)}} \sin \Delta \psi \\
\frac{(a_0 - a(s)) \cos \Delta \psi - (1 + \alpha_0 \alpha(s)) \sin \Delta \psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}} (\cos \Delta \psi - \alpha_0 \sin \Delta \psi)
\end{pmatrix}
\]

- Periodic cell

\[
\mathcal{M}_C = \begin{pmatrix}
\cos \mu + \alpha \sin \mu & \beta \sin \mu \\
-\gamma \sin \mu & \cos \mu - \alpha \sin \mu
\end{pmatrix}
\]

- Mirror symmetric cell

\[
\mathcal{M}_C = \begin{pmatrix}
\cos \mu & \beta \sin \mu \\
-\frac{1}{\beta} \sin \mu & \cos \mu
\end{pmatrix}
\]
Low emittance lattice

LIGHT SOURCE PERFORMANCE

High Brilliance

\[ \tilde{B} \propto \frac{I}{\epsilon_x \epsilon_y} \]

Emittance
Lattice design

Current
RF cavities design
Equilibrium emittance reminder

\[ \epsilon_x = \frac{C_q \gamma^2 \int \mathcal{H}_x(s) \rho_x^3 ds}{\mathcal{J}_x \int \frac{1}{\rho_x^2} ds} \]

\[ C_q = \frac{55 \hbar}{32 \sqrt{3} m_0 c} = 3.83 \times 10^{-13} \text{ m} \]

with the dispersion emittance defined as

\[ \mathcal{H}(s) = \beta(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta'(s) + \gamma(s)\eta(s)^2 \]

- For isomagnetic ring with separated function magnets the equilibrium emittance is written

\[ \epsilon_x = 1470 \frac{E^2}{\rho} \frac{1}{l_{\text{bend}}} \int_0^{l_{\text{bend}}} \mathcal{H}_x(s) ds \]

- Smaller bending angle and lower energy reduce emittance
Twiss functions through a dipole

Consider the transport matrix of a bending magnet (ignoring edge focusing)

\[ M_{\text{sector}} = \begin{pmatrix}
\cos \theta & \rho \sin \theta & \rho (1 - \cos \theta) \\
-\frac{1}{\rho} \sin \theta & \cos \theta & \sin \theta \\
0 & 0 & 1
\end{pmatrix} \]

Consider at its entrance the initial optics functions \( \beta_0, \alpha_0, \gamma_0, \eta_0, \eta_0' \).

The evolution of the twiss functions, dispersion and dispersion derivative are given by

\[
\begin{pmatrix}
\beta(s) \\
\alpha(s) \\
\gamma(s)
\end{pmatrix} = \begin{pmatrix}
\cos \left( \frac{s}{\rho} \right)^2 & -\rho \sin \left( \frac{2s}{\rho} \right) & \rho^2 \sin \left( \frac{s}{\rho} \right)^2 \\
\frac{\sin \left( \frac{2s}{\rho} \right)}{2 \rho} & \cos \left( \frac{2s}{\rho} \right) & -\frac{1}{2} \rho \sin \left( \frac{2s}{\rho} \right) \\
\frac{\sin \left( \frac{s}{\rho} \right)^2}{\rho^2} & \frac{\sin \left( \frac{2s}{\rho} \right)}{\rho} & \cos \left( \frac{s}{\rho} \right)^2
\end{pmatrix} \begin{pmatrix}
\beta(0) \\
\alpha(0) \\
\gamma(0)
\end{pmatrix}
\]

\[
\eta(s) = \eta_0 \cos \left( \frac{s}{\rho} \right) + \eta'_0 \rho \sin \left( \frac{s}{\rho} \right) + \rho (1 - \cos \left( \frac{s}{\rho} \right))
\]

\[
\eta'(s) = -\frac{\eta_0}{\rho} \sin \left( \frac{s}{\rho} \right) + \eta'_0 \cos \left( \frac{s}{\rho} \right) + \sin \left( \frac{s}{\rho} \right)
\]
The dispersion emittance through the dipole is written as

\[ \mathcal{H}(s) = \gamma_0 \left( \eta_0^2 - \frac{1}{2} \eta_0 \left( -4 \rho + 4 \rho \cos \left( \frac{s}{\rho} \right) \right) + \frac{1}{2} \left( 3 \rho^2 - 4 \rho^2 \cos \left( \frac{s}{\rho} \right) + \rho^2 \cos \left( \frac{2s}{\rho} \right) \right) \right) + \beta_0 \left( \frac{1}{2} \left( 1 - \cos \left( \frac{s}{\rho} \right) \right) + 2 \sin \left( \frac{s}{\rho} \right) \eta_0 \right) + (\eta_0')^2 \right) + \alpha_0 \left( \frac{1}{2} \left( -4 \rho \sin \left( \frac{s}{\rho} \right) + 4 \rho \cos \left( \frac{s}{\rho} \right) \sin \left( \frac{s}{\rho} \right) \right) + \frac{1}{2} \left( -4 \rho + 4 \rho \cos \left( \frac{s}{\rho} \right) \right) \eta_0^{'} + \eta_0 \left( 2 \sin \left( \frac{s}{\rho} \right) + 2 \eta_0^{'} \right) \right) \]

and its average along the dipole of length \( l \)

\[ \left\langle \mathcal{H}(s) \right\rangle = \gamma_0 \left( \eta_0^2 - \frac{2 \eta_0 \rho \left( 1 - \rho \sin \left( \frac{1}{\rho} \right) \right)}{1} + \rho^2 \left( 6 - 8 \rho \sin \left( \frac{1}{\rho} \right) + \rho \sin \left( \frac{2}{\rho} \right) \right) \right) + \beta_0 \left( \frac{1}{2} - \frac{\rho \sin \left( \frac{2}{\rho} \right)}{4} - \frac{2 \rho \left( -1 + \cos \left( \frac{1}{\rho} \right) \right)}{1} \right) + (\eta_0')^2 \right) + \alpha_0 \left( \frac{4 \rho^2 \sin \left( \frac{1}{2} \right)^4}{1} - \frac{2 \rho \left( 1 - \rho \sin \left( \frac{1}{\rho} \right) \right)}{1} \eta_0' + \frac{2 \eta_0 \left( \rho - \rho \cos \left( \frac{1}{\rho} \right) + 1 \eta_0' \right)}{1} \right) \]
Optics functions for minimum emittance

- Take the derivative of the dispersion emittance with respect to the initial optics functions and equate it to zero to find the minimum conditions

- Non-zero dispersion (general case)

\[
\beta_0 = \frac{\rho^3 \left(2 \left(-1 + \Theta^2 + \cos (2 \Theta)\right) + \Theta \sin (2 \Theta)\right)}{\sqrt{2} \sqrt{\Theta} \rho^4 \left(-9 \Theta + 2 \Theta^3 + 8 \Theta \cos [\Theta] + \Theta \cos (2 \Theta) + 8 \sin [\Theta] - 4 \sin (2 \Theta)\right)}
\]

\[
\alpha_0 = \frac{\rho^2 \left(-\Theta + \Theta \cos (2 \Theta) + 4 \sin [\Theta] - 2 \sin (2 \Theta)\right)}{\sqrt{2} \sqrt{\Theta} \rho^4 \left(-9 \Theta + 2 \Theta^3 + 8 \Theta \cos [\Theta] + \Theta \cos (2 \Theta) + 8 \sin [\Theta] - 4 \sin (2 \Theta)\right)}
\]

\[
\eta_0 = \rho - \frac{\rho \sin [\Theta]}{\Theta} \quad \text{and} \quad \eta_0' = \frac{-1 + \cos [\Theta]}{\Theta}
\]

- Zero dispersion (and its derivative)

\[
\beta_0 = \frac{\rho^2 \left(6 \Theta - 8 \sin [\Theta] + \sin (2 \Theta)\right)}{\sqrt{2} \sqrt{-\rho^2} \left(9 - 6 \Theta^2 - 16 \cos [\Theta] + 7 \cos (2 \Theta) + 8 \Theta \sin [\Theta] + 2 \Theta \sin (2 \Theta)\right)}
\]

\[
\alpha_0 = \frac{4 \rho \sin \left(\frac{\Theta}{2}\right)^4}{\sqrt{-\frac{9 \rho^2}{2} + 3 \Theta^2 \rho^2 - \frac{1}{2} \rho \left(-16 \rho \cos [\Theta] + 7 \rho \cos (2 \Theta) + 2 \Theta \rho \left(4 \sin [\Theta] + \sin (2 \Theta)\right)\right)}}
\]
In the general case, the equilibrium emittance takes the form

\[ \epsilon_x = \frac{1}{J_x \theta^2} \left( 735 \sqrt{2} \, \text{En}^2 \, \sqrt{\theta (-9 + 2 \theta^3 + 8 \theta \cos \theta + \theta \cos 2\theta + 8 \sin \theta - 4 \sin 2\theta}) \right) \]

and expanding on \( \theta \) we have

\[ \epsilon_x = \frac{49 \sqrt{\frac{5}{3} \, \text{En}^2 \theta^3}}{2 \, J_x} - \frac{7 \left( \sqrt{\frac{3}{5} \, \text{En}^2} \right) \theta^5}{4 \, J_x} + O(\theta^6) \]

In the 0-dispersion case,

\[ \epsilon_x = \frac{735 \sqrt{2} \, \text{En}^2 \sqrt{-9 + 6 \theta^2 + 16 \cos \theta - 7 \cos 2\theta - 4 \theta (2 + \cos \theta) \sin \theta}}{J_x \theta} \]

The second order term is negligible (less the 1% for \( \theta < 20 \) deg.)

Note that in both cases the emittance depends on the 3rd power of the bending angle

The emittance for non-zero dispersion is 3 times smaller
Optics functions

\[
\beta_0 = \frac{8 \rho \theta}{\sqrt{15}} - \frac{4 \rho \theta^3}{5 \sqrt{15}} + \frac{257 \rho \theta^5}{7350 \sqrt{15}} + O[\theta]^6
\]

\[
\alpha_0 = \sqrt{15} - \frac{4}{7} \sqrt{\frac{3}{5}} \theta^2 + \frac{269 \theta^4}{1470 \sqrt{15}} + O[\theta]^6
\]

\[
\eta_0 = \frac{\rho \theta^2}{6} - \frac{\rho \theta^4}{120} + O[\theta]^6
\]

\[
\eta_0' = -\frac{\theta}{2} + \frac{\theta^3}{24} - \frac{\theta^5}{720} + O[\theta]^6
\]

\[
\beta_0 = 2 \sqrt{\frac{3}{5}} \rho \theta - \frac{2 \rho \theta^3}{5 \sqrt{15}} + \frac{11 \sqrt{\frac{3}{5}} \rho \theta^5}{2450} + O[\theta]^6
\]

\[
\alpha_0 = \sqrt{15} - \frac{4}{7} \sqrt{\frac{3}{5}} \theta^2 + \frac{13}{490} \sqrt{\frac{3}{5}} \theta^4 + O[\theta]^6
\]

\[
\eta_0 = \eta_0' = 0
\]
Deviation from the minimum emittance

- Introduce the dimensionless quantities
  \[ \eta_0 \] and \[ F = \frac{\epsilon_x}{\epsilon_{x;\text{min}}} \]

- Introduce them into the expression of the mean dispersion emittance to get
  \[ F^2 = (\bar{\beta}_0 - F)^2 + 5/4(\eta_0 - 1)^2 \]

- The curves of equal relative emittance are ellipses

- The phase advance for a mirror symmetric cell is
  \[ \mu = \arctan \left( \frac{6}{\sqrt{15}} \frac{\bar{\beta}_0}{\eta_0 - 3} \right) \]

- The optimum phase advance for reaching the absolute minimum emittance (F=1) is unique (284.5°)!
Effective emittance

Horizontal dispersion in the straight section \( \eta_x \neq 0 \)

Reaching the minimum theoretical emittance

Enlargement of the beam size through the electron energy spread at the ID

The brilliance \( \tilde{B} \propto \frac{I}{\epsilon_{x,eff}(s_{ID})\epsilon_{y,eff}(s_{ID})} \) is inversely proportional to the effective emittance \( \epsilon_{x,eff}(s)^2 \equiv \langle x(s)^2 \rangle \langle x'(s)^2 \rangle - \langle x(s)x'(s) \rangle^2 \).

After replacing the expressions for position and angles and consider that the alpha function and dispersion derivative are zero on the ID

\[
\epsilon_{x,eff}(s_{ID}) = \sqrt{\epsilon_x^2 + H_x(s_0)\epsilon_x \sigma_\delta^2}
\]
Low emittance lattices

- Double Bend Achromat (DBA)
- Triple Bend Achromat (TBA)
- Quadruple Bend Achromat (QBA)
- Minimum Emittance Lattice (MEL)
Dispersion suppressors

- Dispersion has to be eliminated in special areas like injection, extraction or interaction points (orbit independent to momentum spread)

- Use dispersion suppressors
  - Eliminate two dipoles in a FODO cell (missing dipole)
  - Set last dipoles with different bending angles
    \[
    \theta_1 = \theta \left(1 - \frac{1}{4\sin^2 \mu_{HFODO}}\right)
    \]
    \[
    \theta_2 = \frac{\theta}{4\sin^2 \mu_{HFODO}}
    \]
  - For equal bending angle dipoles the FODO phase advance should be equal to \(\pi/2\)
Chasmann-Green cell

- Double bend achromat with unique central quadrupole
- Achromatic condition is assured by tuning the central quadrupole
- Minimum emittance with a quadrupole doublet in either side of the bends
- The required focal length of the quad is given by
  \[ f = \frac{1}{2} (L_{\text{drift}} + \frac{1}{2} L_{\text{bend}}) \]
  and the dispersion
  \[ D_c = (L_{\text{drift}} + \frac{1}{2} L_{\text{bend}}) \theta \]
- Disadvantage the limited tunability and reduced space
Central triplet between the two bends and two triplets in the straight section to achieve the minimum emittance and achromatic condition

- Elettra (Trieste) uses this lattice achieving almost the absolute minimum emittance for an achromat
- Disadvantage the increased space in between the bends
Expanded DBA

- Original lattice of ESRF storage ring, with 4 quadrupoles in between the bends
- Alternating moderate and low beta in intertions
Original lattice of ESRF storage ring, with 4 quadrupoles in between the bends

- Alternating moderate and low beta in insertions
General double bend structure

- Reduce emittance by allowing dispersion in the straight sections
- ESRF reduced emittance almost halved the emittance achieved

\[ \varepsilon_{x0} = 4 \, \text{nm.rad} \ @ \ 6\text{GeV} \quad C = 844\text{m} \]
Theoretical minimum emittance optics

- Old Super-Aco ring could operate in a theoretical minimum emittance optics
- Structure mostly used in damping rings

\[ \nu_x = 5.86 \]
\[ \nu_z = 2.86 \]
\[ \varepsilon_x = 9.85 \text{ nm.rad} @ 800 \text{ MeV} \]
Triple Bend Achromat

- Three bends with the central one with theoretical minimum emittance conditions

- Strict relationship between the bending angles and lengths of dipoles in order to achieve dispersion matching: \[ \frac{L_2^3}{\rho_2^2} = 3 \frac{L_1^3}{\rho_1^2} \]

- A unique phase advance of 255° is needed for reaching the minimum emittance

- This minimum is equal to the one of the DBA

- Example, the Swiss Light Source

\[ \varepsilon_{x0} = 5 \text{nm.rad} @ 2.4 \text{GeV} \]
## Light sources performance

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (GeV)</th>
<th>$\Theta$</th>
<th>C(m)</th>
<th>$\Sigma L_{SS}$</th>
<th>$\varepsilon x_0$ (nm.rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALS</td>
<td>1.9</td>
<td>0.1745</td>
<td>197</td>
<td>81</td>
<td>5.6</td>
</tr>
<tr>
<td>BESSYII</td>
<td>1.9</td>
<td>0.1963</td>
<td>240</td>
<td>89</td>
<td>6.4</td>
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<tr>
<td>DIAMOND</td>
<td>3</td>
<td>0.1309</td>
<td>562</td>
<td>218.2</td>
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<tr>
<td>ESRF</td>
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<td>0.09817</td>
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<tr>
<td>ELETTRA</td>
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<td>258</td>
<td>74.78</td>
<td>7</td>
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<tr>
<td>SLS</td>
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<td>0.2440</td>
<td>288</td>
<td>63</td>
<td>5</td>
</tr>
<tr>
<td>SOLEIL</td>
<td>2.75</td>
<td>0.1963</td>
<td>354</td>
<td>159.6</td>
<td>3.7</td>
</tr>
</tbody>
</table>
Circumference and periodicity

- Circumference choice implicates
  - Tunnel length should be small to reduce cost
  - Optics constraints necessitate circumference increase
  - Available spaces should not be reduce for all necessary equipment to fit
  - Sometimes it should be a multiple of the RF harmonic number and the RF wavelength
  - Varies from a few 1m to 27km (LEP)

- Large Periodicity implies
  - Simplicity in design and operation
  - Stability for dangerous resonance crossing (avoid only structural ones)
  - Reduction of cost for a few types of magnets
  - Varies from 1 (DORIS) to 40 (APS)
In a ring, the **tune** is defined from the 1-turn phase advance

\[ Q_{x,y} = \frac{1}{2\pi} \oint ds / \beta_{x,y}(s) \]

i.e. number betatron oscillations per turn

Taking the average of the betatron tune around the ring we have in **smooth approximation**

\[ 2\pi Q = \frac{C}{\langle \beta \rangle} \rightarrow Q = \frac{R}{\langle \beta \rangle} \]

Extremely useful formula for deriving scaling laws

The position of the tunes in a diagram of horizontal versus vertical tune is called a **working point**

The tunes are imposed by the choice of the quadrupole strengths

One should try to avoid **resonance conditions**
Ideal versus real lattice


A. Wolski, Low emittance machines, CERN Accelerator School, September 2007.
