Synchrotron Radiation

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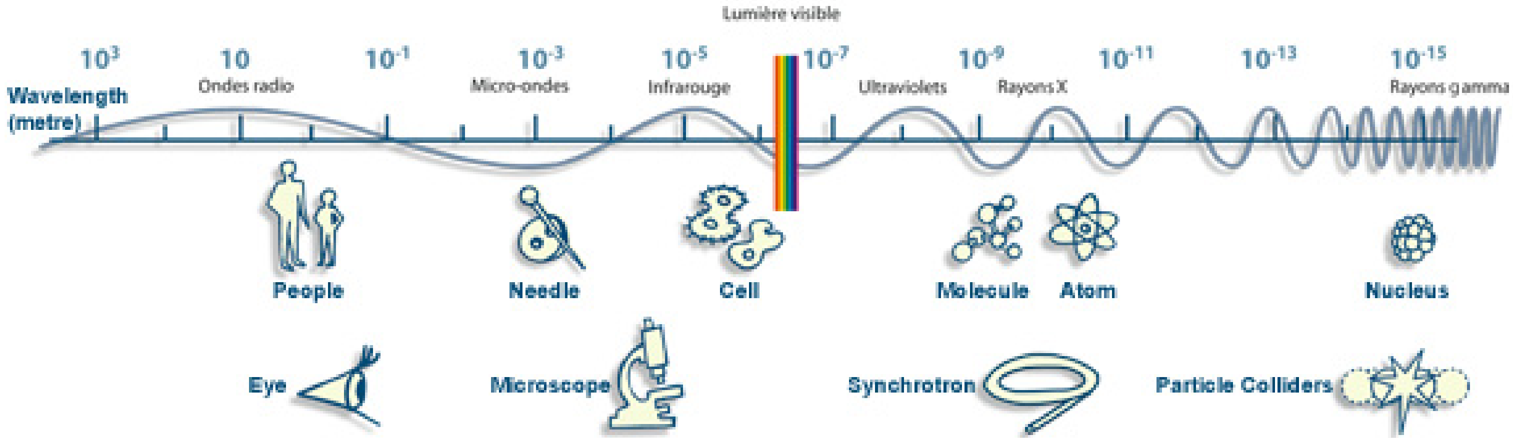
Synchrotron light

Röntgen, 1895

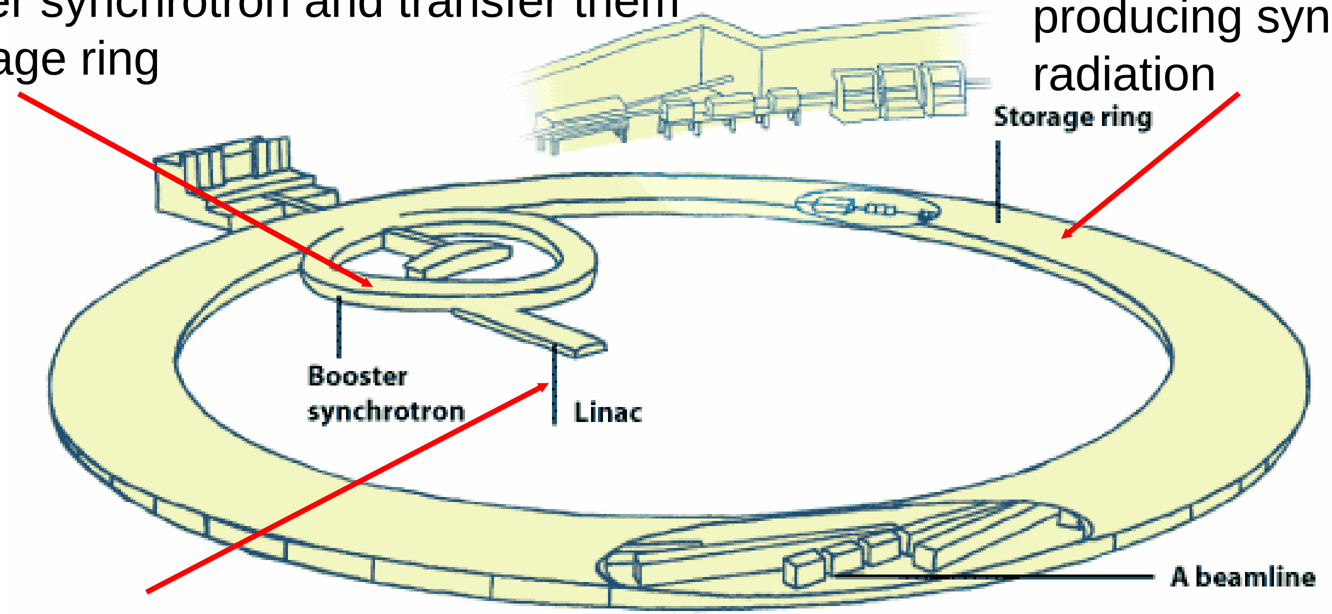
J. P. Blewett, *Phys. Rev*. **69**, 87 (1946);
F. R. Elder, R. V. Langmuir, A. M. Gurewitsch, H. C. Pollock, *Phys. Rev.* **71**, 827 (1947)



Creating the light

Electrons are getting accumulated up to a high current (a few hundred mA) in the storage ring and they circulate freely producing synchrotron radiation

Accelerate electrons up to a few GeV in a few msec in the booster synchrotron and transfer them into the storage ring

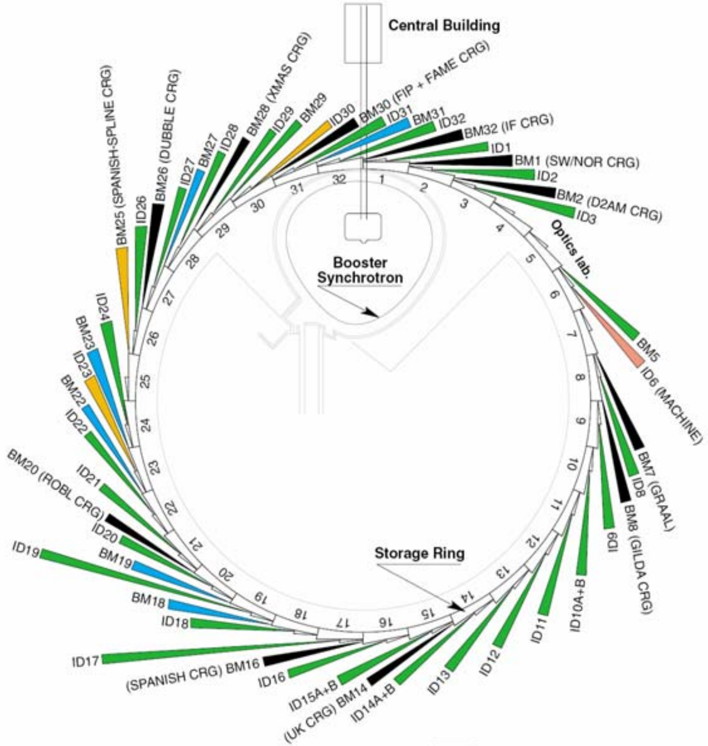


Produce electron in a thermionic gun, accelerate them up to a few MeV in a linac and transfer them into a booster

Procedure repeated periodically, depending on the beam lifetime

A typical storage ring – the ESRF

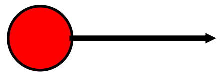
- The **first** and **most brilliant** 3rd generation light source in Europe
- **50 beam lines** collecting X-rays from insertion devices and bending magnets
- **3500 users/year** from **14 member countries** carrying X-ray spectroscopy experiments for material science, chemistry, biology, medicine, earth sciences, archeology, etc.
- The machine comprises an **e$^-$ linac**, a 300m-**booster** and an 844m-**storage ring**
- The storage ring has a record **availability of 98%** with a **mean-time between failures** of more than **2 days**



ESRF parameters

Energy	GeV	6.03
Maximum Current	mA	200
Horizontal Emittance	nm	4
Vertical Emittance (*minimum achieved)	nm	0.025 (0.010*)
Coupling (*minimum achieved)	%	0.6 (0.25*)
Revolution frequency	kHz	355
Number of bunches		1 to 992
Time between bunches	ns	2816 to 2.82

Why circular machines?



$$\mathbf{p} = m_0 \mathbf{v} \qquad v << c$$

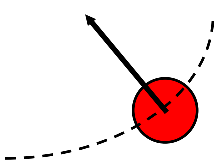
$$P_s = \frac{e^2}{6\pi\varepsilon_0 m_0^2 c^3} \left(\frac{d\mathbf{p}}{dt} \right)^2$$

Larmor Power radiated by non-relativistic particles is very small

$$\mathbf{p} = \gamma m_0 \mathbf{v} \qquad v \approx c$$

$$P_s = \frac{e^2}{6\pi\varepsilon_0 m_0^2 c^3} \left(\frac{dp}{dt} \right)^2$$

Power radiated by relativistic particle in linear accelerator is negligible



$$P_s = \frac{e^2 c}{6\pi\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho^2}$$

Power radiated by relativistic particle in circular accelerator is very strong (Liénard, 1898)

Why electrons?

$$P_s = \frac{e^2 c}{6\pi\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho^2}$$

Power inversely proportional to 4th power of rest mass (proton **1000 times** heavier than electron)
On the other hand, for multi TeV hadron colliders (LHC) synchrotron radiation is an important issue (protection with absorbers)

$$\Delta E = \frac{e^2}{3\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho}$$

$$\Delta E[keV] = 88.5 \frac{E^4[GeV^4]}{\rho[m]}$$

By integrating around one revolution we get the **energy loss per turn.**
For the ESRF is around 5 MeV/turn.
On the other hand, for LEPII (**120 GeV**) it was 6GeV/turn, i.e. circular electron machines of more than 100GeV are not practical

Properties of Synchrotron Radiation

- **Time compression:** the emitted wavelength is compressed

$$\lambda_o = \frac{1}{\gamma^2}\lambda_e$$

- **Angular collimation:** emission angle is also reduced

$$\theta_o = \frac{1}{\gamma}\theta_e$$

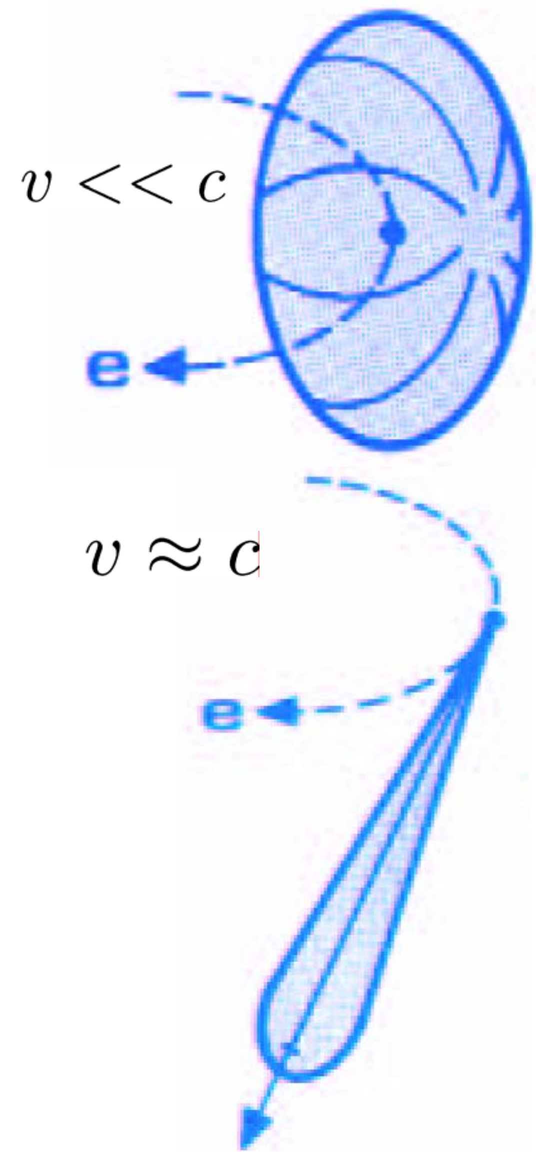
- **Short emission pulse:** time difference between first and last emitted photons very short. Radiation signal can be produced with a time structure

$$\Delta t \approx \frac{4\rho}{3c\gamma^3}$$

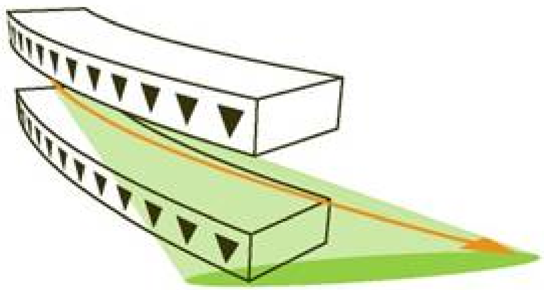
- **Continuous frequency spectrum:** very high harmonics of the revolution frequency

$$\omega_c \approx \gamma^3 \omega_0$$

- **Polarization:** horizontal in the plane of the particle orbit, elliptical in general (background scattering minimized, sensitivity improved)



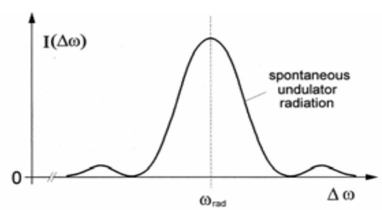
Magnetic elements for light production



Bending magnet (Sweeping searchlight)

At each deflection of the electron path a beam of radiation is produced.

Insertion devices

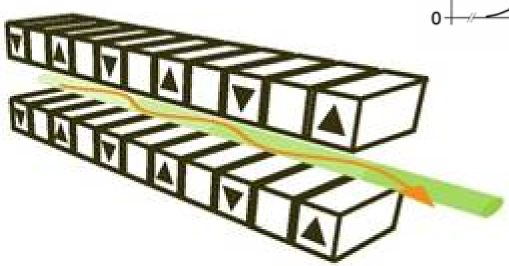


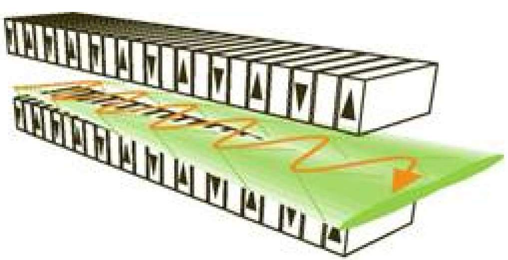
Undulator / wiggler parameter

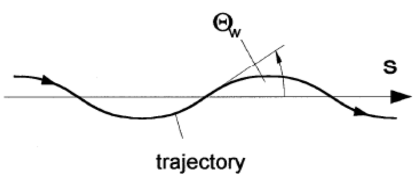
$$K = \frac{\lambda_u e \tilde{B}}{2\pi m_e c}$$

Deflection angle

$$\Theta_W = \frac{K}{\gamma}$$







Undulator (K≤1)

Produces a very narrow beam of coherent light

Wiggler (K>1)

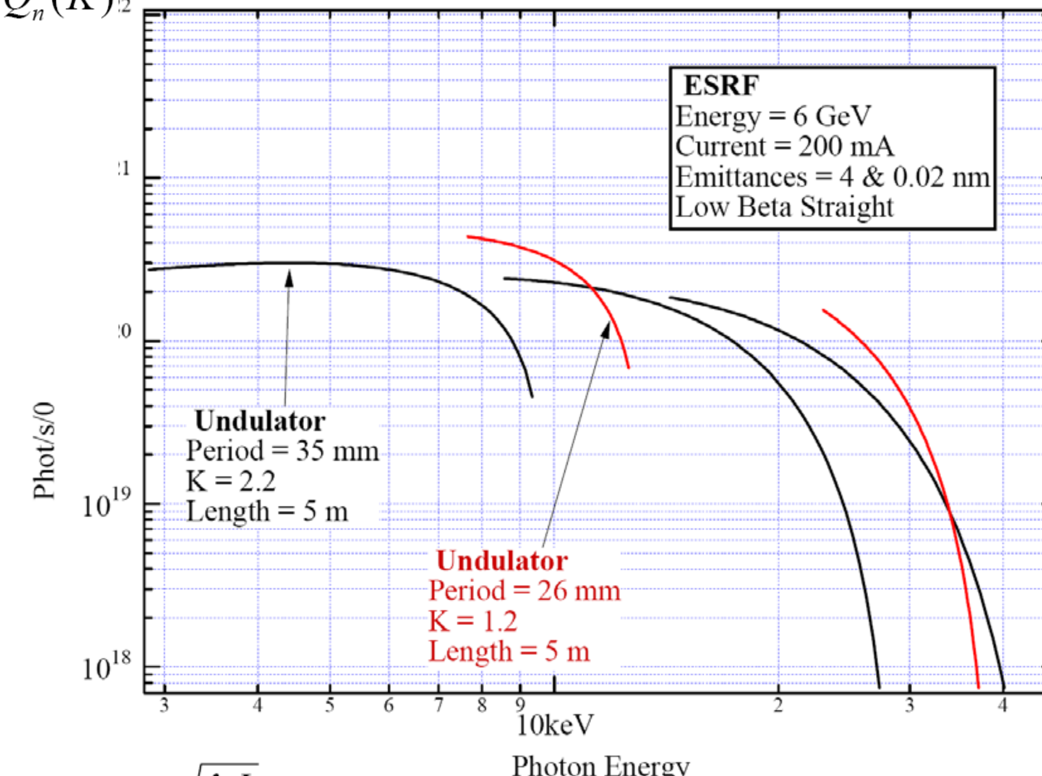
Beams emitted at each pole reinforce each other and appear as a broad beam of incoherent light.

$$\frac{dF_n}{\frac{d\lambda}{\lambda}} = \int \frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(\theta_x,\theta_z,\lambda_n)\, d\theta_x d\theta_z = \pi\alpha \frac{I}{e} N Q_n(K)$$

$$\boxed{F_n\,[Ph/\mathrm{sec}/0.1\%] = 1.431\,10^{14}\ N\, I[A]\, Q_n(K)}$$

- N : Number of Undulator Periods
- I : Ring Current
- n : Harmonic Number

Maximum angular spectral flux



Spectral brilliance and brightness

$$\boxed{B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}}$$

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_R^2}\ , \qquad \sigma_R = \frac{\sqrt{\lambda_n L}}{2\pi}$$

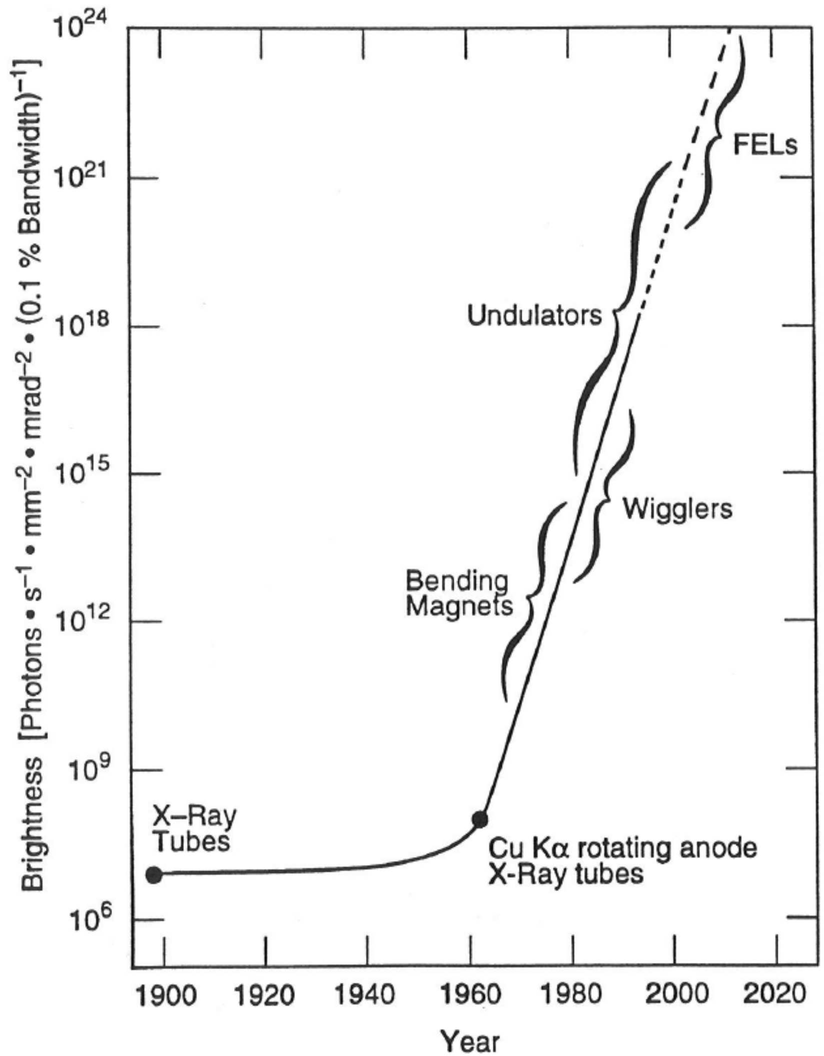
$$\Sigma'_x = \sqrt{\sigma'^2_x + \sigma'^2_R}\ , \qquad \sigma'_R = \frac{\lambda_n}{2L}$$

Σ_x, Σ_z : Photon beam sizes

Σ'_x, Σ'_z: Photon beam divergences

Synchrotron Radiation Sources

- **X-ray tubes** (early 20th century)
- **1st generation:** originally build for high-energy physics experiments and synchrotron radiation programs used parasitically
- **2nd generation:** dedicated synchrotron sources based on bending magnets
- **3rd generation:** synchrotron radiation is produced in undulators and wigglers
- **4th generation:** free electron lasers

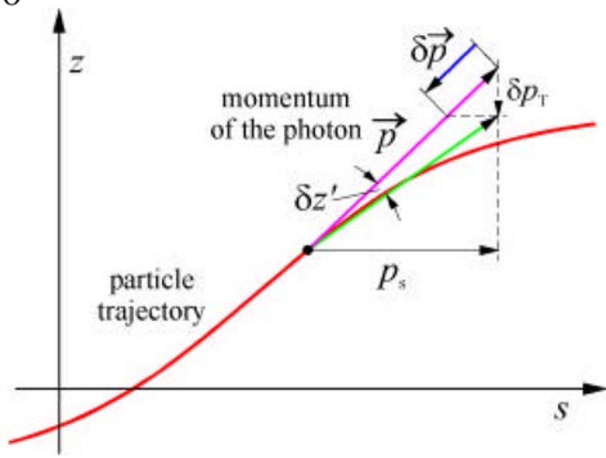


Beam Lifetime

- Aperture
  - Physical (magnets, vacuum chambers)
  - Dynamic (non-linear errors, chromaticity sextupoles)
  - Off-momentum (transverse and longitudinal momentum acceptance)
- Beam-Gas interaction
  - Elastic scattering (electron loss)
  - Inelastic scattering (Bremsstrahlung, energy transfer)
  - Ion trapping (pressure increase, emittance blow-up)
- Intra-beam scattering
  - Large angle collisions - Tousckek lifetime (momentum transfer)
  - Small angle collisions
- Quantum lifetime (Gaussian distribution within a physical aperture)
- Increasing Lifetime
  - NEG coating
  - Top-up operation (continuous injection)
  - Increasing the bunch length (harmonic cavities)

Damping of oscillations

- Synchrotron oscillation
  - The energy is damped following the equations $\Delta\ddot{E} + \alpha_s \Delta\dot{E} + \Omega\Delta E = 0$ with damping coefficient and
$$\alpha_s = \frac{1}{2T_0}\frac{dW}{dE} = \frac{W}{2ET_0}(2+D)$$
  - The lost energy is recovered by the RF



- Betatron oscillations
  - The vertical oscillation is damped with a coefficient
$$\alpha_y = \frac{W}{2ET_0}$$
  - The horizontal oscillation is damped with a coefficient
$$\alpha_x = \frac{W}{2ET_0}(1-D)$$
  - The vertical oscillations are damped to 0 (ideally)
  - The horizontal oscillations reach an equilibrium due to quantum fluctuations of photon emission
- Robinson criterion $J_x = 1 - \frac{\oint \frac{\eta_x(s)}{\rho_x^3}(1+2k\rho_x^2)ds}{\oint \frac{1}{\rho_x^2}ds}$, $J_y = 1$, $J_x + J_y + J_s = 4$

LIGHT SOURCE QUALITY FACTOR

Increase the **Brilliance** by an order of magnitude

$$\tilde{B} \propto \frac{I}{\epsilon_x \epsilon_y}$$

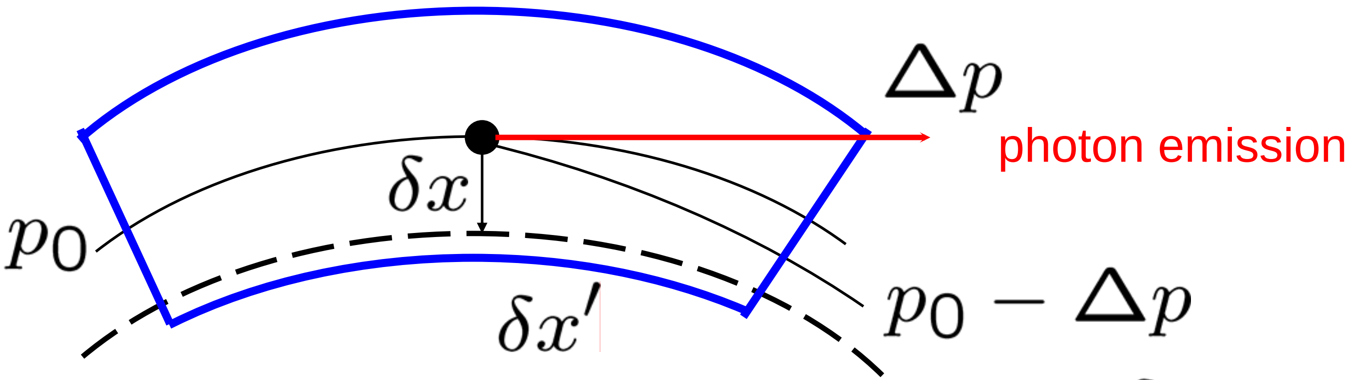
Current

New RF cavities design

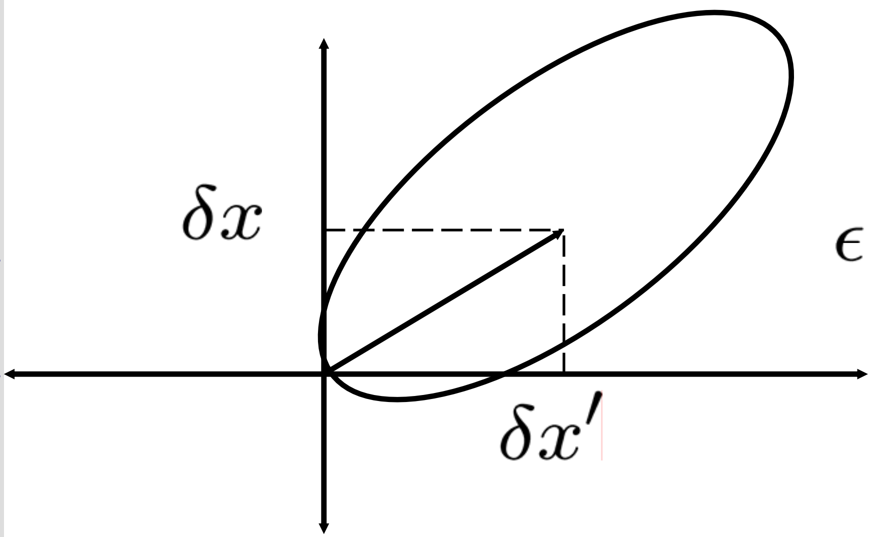
Emittance

New lattice design

Equilibrium emittance



$$\delta x = \eta \frac{\delta_p}{p} \quad \text{and} \quad \delta x' = \eta' \frac{\delta_p}{p}$$



$$\epsilon = \frac{\delta_p}{p} \mathcal{H}(s)$$

with

$$\mathcal{H} = \beta \eta'^2 + 2\alpha \eta \eta + \gamma \eta^2$$

After averaging all over the ring

$$\epsilon_x = \frac{C_q \gamma^2}{J_x} \frac{\oint \frac{\mathcal{H}_x(s)}{|\rho_x|^3} ds}{\oint \frac{1}{\rho_x^2} ds}$$

Minimum emittance conditions

- Consider the transport matrix of a bending magnet (ignoring edge focusing)

$$\begin{pmatrix} \cos\left(\frac{s}{\rho}\right) & \rho \sin\left(\frac{s}{\rho}\right) & \rho\left(1-\cos\left(\frac{s}{\rho}\right)\right) \\ -\frac{\sin\left(\frac{s}{\rho}\right)}{\rho} & \cos\left(\frac{s}{\rho}\right) & \sin\left(\frac{s}{\rho}\right) \\ 0 & 0 & 1 \end{pmatrix}$$

- Consider at its entrance the initial optics functions $\beta_0, \alpha_0, \gamma_0, \eta_0, \eta_0'$ and their evolution along the magnet is given by

$$\beta = \beta_0 \cos^2\left(\frac{s}{\rho}\right) - \alpha_0 \rho \sin\left(\frac{2s}{\rho}\right) + \gamma_0 \rho^2 \sin^2\left(\frac{s}{\rho}\right)$$

$$\alpha = \alpha_0 \cos\left(\frac{2s}{\rho}\right) - \frac{(\beta_0 - \gamma_0 \rho^2)\sin\left(\frac{2s}{\rho}\right)}{2\rho}$$

$$\gamma = \gamma_0 \cos^2\left(\frac{s}{\rho}\right) + \frac{\beta_0 \sin^2\left(\frac{s}{\rho}\right) + \alpha_0 \rho \sin\left(\frac{2s}{\rho}\right)}{\rho^2}$$

$$\eta = \rho + (\eta_0 - \rho)\cos\left(\frac{s}{\rho}\right) + \eta_0' \rho \sin\left(\frac{s}{\rho}\right)$$

$$\eta' = \frac{-(\eta_0 - \rho)\sin\left(\frac{s}{\rho}\right)}{\rho} + \cos\left(\frac{s}{\rho}\right)\eta_0'$$

Computing the invariant

- The average dispersion emittance along the bending magnet is

$$
\begin{aligned}
\langle H \rangle = \; & \gamma_0 \left(\ldots - \frac{2\eta_0 \rho \left(\ldots - \rho \sin\left(\frac{L}{\rho}\right) \ldots \right)}{L} \ldots \frac{\ldots L - 8\rho \sin\left(\frac{L}{\rho}\right) \ldots \sin\left(\frac{L}{\rho}\right) \ldots}{4L} \right) \\
& \beta_0 \left(\frac{1}{2} - \frac{\rho \sin\left(\frac{2L}{\rho}\right)}{4L} \ldots \frac{\ldots \left(1 + \cos\left(\frac{L}{\rho}\right) \right) \ldots}{L} + \ldots \eta_0' \ldots \right) \\
& \alpha_0 \left(\frac{4\rho^2 \sin\left(\frac{L}{2\rho}\right) \ldots}{L} - \frac{\ldots \rho \left(\ldots - \rho \sin\left(\frac{L}{\rho}\right) \right) \ldots}{L} + \frac{\ldots \eta_0 \left(\ldots - \rho \cos\left(\frac{L}{\rho}\right) \right) \ldots \eta_0'}{L} \right)
\end{aligned}
$$

- Take the derivatives with respect to the optics functions in order to compute the ones minimizing the emittance

Compute the optics functions

- Distinguish two cases:
  - Non-zero dispersion (general case)

$$b0 = \frac{[illegible]\, q + [illegible]\, \mathrm{Cos}(2q)\, [illegible]}{2\, r\, q\, [illegible]\, 9q - 2q^3 + 8q\,\mathrm{Cos}(q)\, [illegible]\, \mathrm{Cos}(2q)\, [illegible]\, \mathrm{Sin}(q)\, [illegible]\, \mathrm{Sin}(2q)\, [illegible]}$$

$$a0 = \frac{[illegible]^2 + \mathrm{Cos}\, [illegible]}{2\, q\, [illegible]\, 9q - 2q^3 + 8q\,\mathrm{Cos}(q)\, [illegible]\, \mathrm{Cos}(2q)\, [illegible]\, \mathrm{Sin}(q)\, [illegible]\, \mathrm{Sin}(2q)\, [illegible]}$$

$$h0 = r - \frac{r\,\mathrm{Sin}(q)}{q} \qquad h0' = \frac{-1 + \mathrm{Cos}(q)}{q}$$

  - Initial dispersion and its derivative equal to 0

$$b0 = \frac{r\, [illegible]\, \mathrm{Sin}\, [illegible]}{2\, [illegible]\, -9 + 6q^2 + 16\,\mathrm{Cos}(q)\, [illegible]\, \mathrm{Cos}(2q)\, [illegible]\, \mathrm{Sin}(q)\, [illegible]\, \mathrm{Sin}(2q)\, [illegible]}$$

$$a0 = \frac{[illegible]\,\mathrm{Sin}\, [illegible]}{2\, [illegible] + 3q^2 + 8\,\mathrm{Cos}(q)\, [illegible]\, \mathrm{Cos}(2q)\, [illegible]\, \mathrm{Sin}(q)\, [illegible]\, \mathrm{Sin}(2q)\, [illegible]}$$

Minimum emittance conditions

- In the general case, the equilibrium emittance takes the form

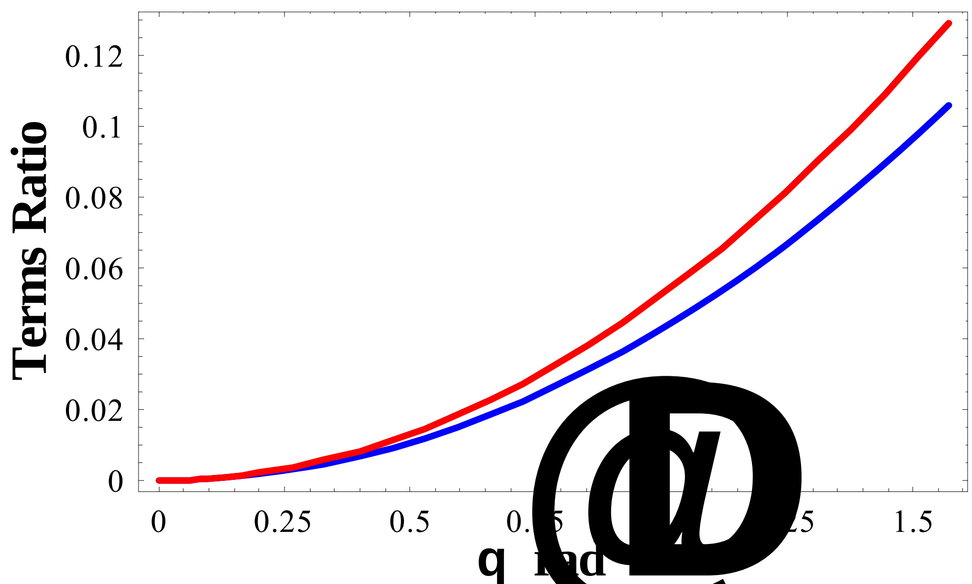
$$\epsilon_x = \frac{C_q \gamma^2}{J_x \sqrt{2}\,\theta^2}\; \theta\,[9\theta\,\ldots\,2\theta^3 + 8\theta\cos(\theta)\,\ldots\cos(2\theta)\,\ldots\sin(\theta)\,\ldots\sin(2\theta)\,\ldots$$

and expanding on θ we have

$$\epsilon_x = \frac{C_q \gamma^2}{J_x}\left(\frac{\theta^3}{12\sqrt{15}} - \frac{\theta^5}{280\sqrt{15}} + O(\theta^6)\right)$$

- In the 0-dispersion case,

$$\epsilon_x = \frac{C_q \gamma^2}{J_x \sqrt{2}\,\theta}\; -9 + 6\theta^2 + 16\cos(\theta)\,\ldots\cos(2\theta)\,\ldots + \cos(\theta)\,\ldots$$

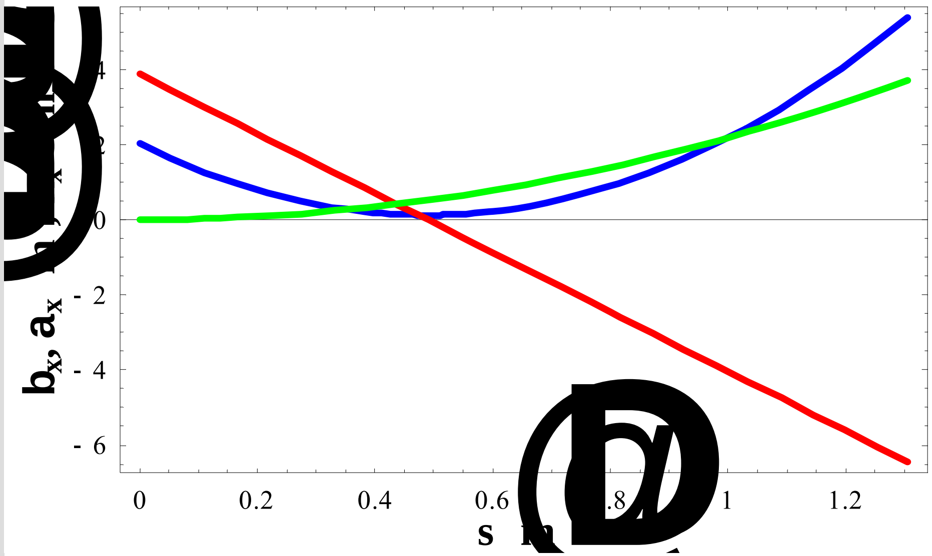


and

$$\epsilon_x = \frac{C_q \gamma^2}{J_x}\left(\frac{\theta^3}{4\sqrt{15}} - \frac{11\,\theta^5}{840\sqrt{15}} + O(\theta^6)\right)$$

- The second order term is negligible

(less the 1% for θ < 20 deg.)

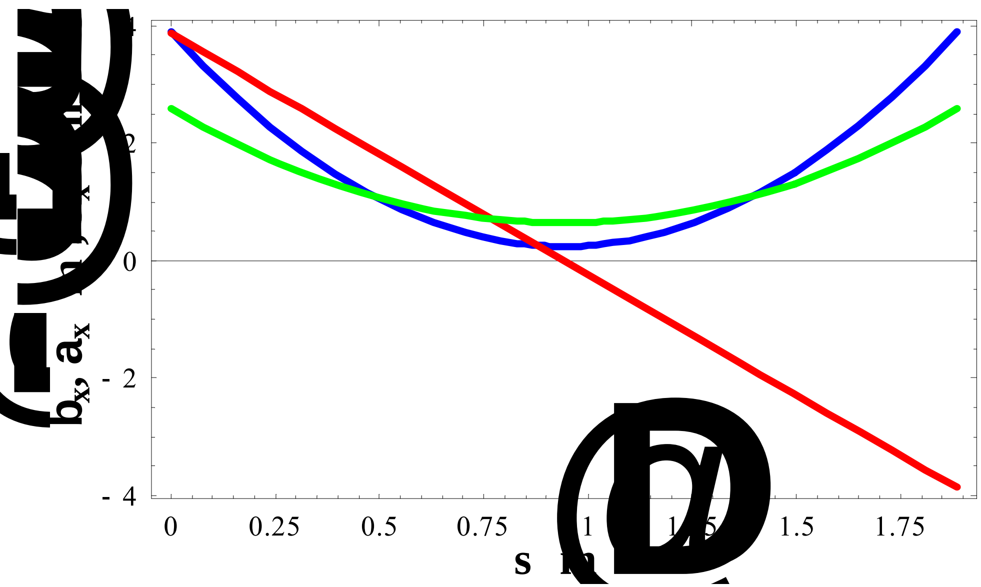


$$\beta_0 = 2\sqrt{\frac{3}{5}}\,\rho\,\theta - \frac{2\,\rho\,\theta^3}{5\sqrt{15}} + \frac{11\sqrt{\frac{3}{5}}\,\rho\,\theta^5}{2450} + O(\ldots)$$

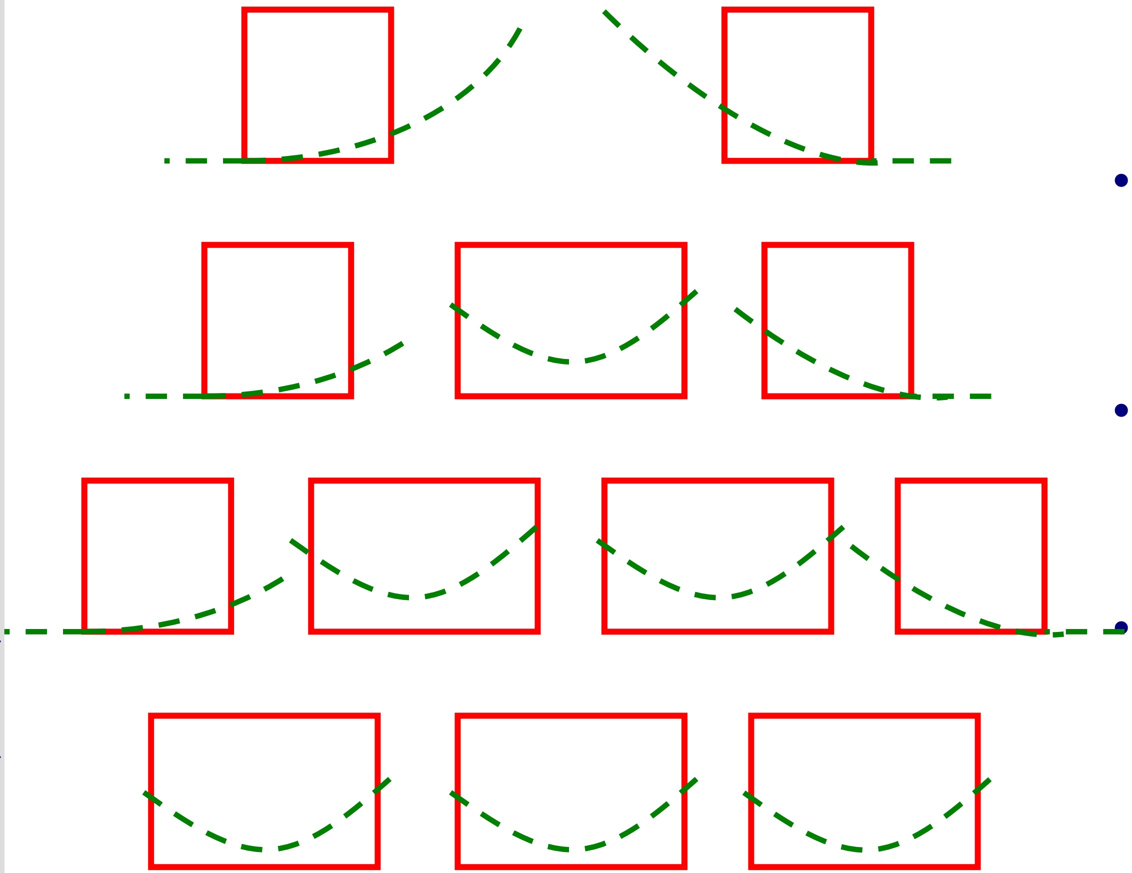
$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{3}{5}}\,\theta^2 + \frac{13}{490}\sqrt{\frac{3}{5}}\,\theta^4 + O(\ldots)$$

$$\beta_0 = \frac{8\,\rho\,\theta}{\sqrt{15}} - \frac{4\,\rho\,\theta^3}{5\sqrt{15}} + \frac{257\,\rho\,\theta^5}{7350\sqrt{15}} + O(\ldots)$$

$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{5}{3}}\,\theta^2 + \frac{269\,\theta^4}{1470\sqrt{15}} + O(\ldots)$$

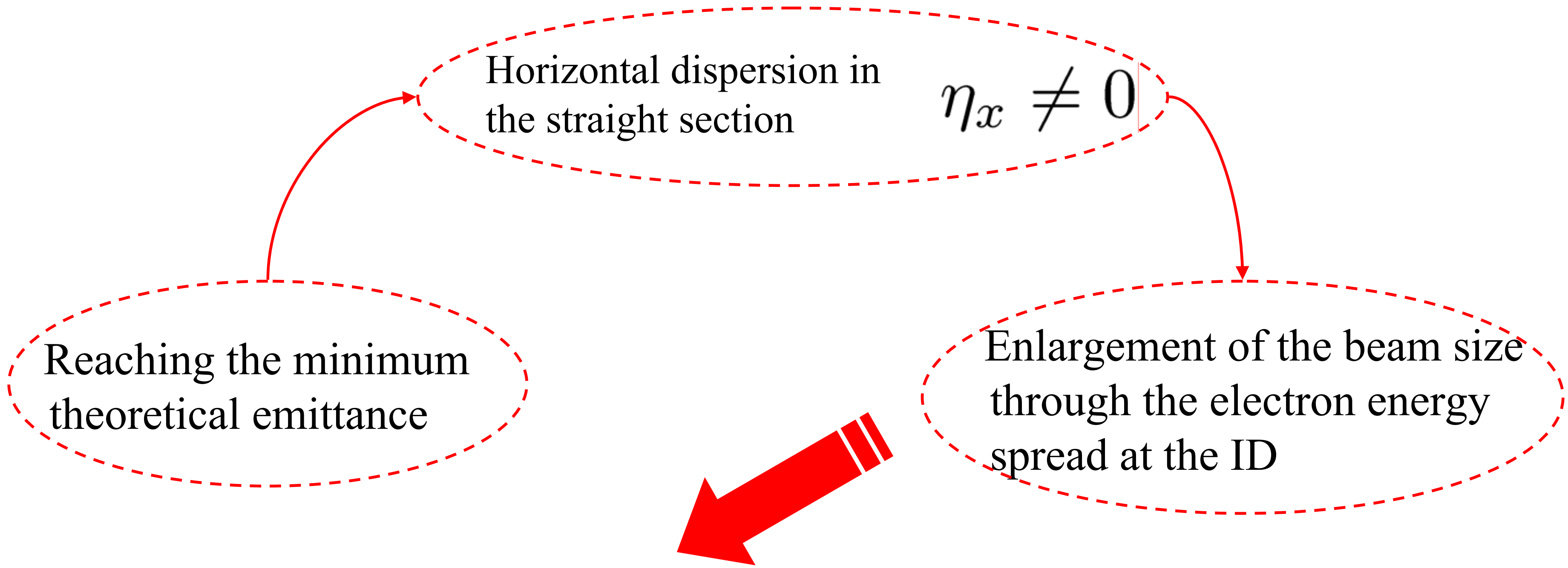


Low emittance lattices



- Double Bend Achromat (DBA)
- Triple Bend Achromat (TBA)
- Quadruple Bend Achromat (QBA)
- Minimum Emittance Lattice (MEL)

The notion of effective emittance



The brilliance $\tilde{B} \propto \dfrac{I}{\epsilon_{x_{eff}}(s_{ID})\epsilon_{y_{eff}}(s_{ID})}$

is inversely proportional to the **effective emittance**

$$\epsilon_{x_{eff}}(s_{ID}) = \sqrt{\epsilon_x^2 + \mathcal{H}_x(s_0)\epsilon_x\sigma_\delta^2}$$

Effective emittance and radiation integrals

$$\epsilon_{x_{eff}}(s_{ID}) = \sqrt{\epsilon_x^2 + \mathcal{H}_x(s_0)\epsilon_x\sigma_\delta^2}$$

$$\mathcal{H}_x(s) = \beta_x(s)\eta_x'^2(s) + 2\alpha_x(s)\eta_x(s)\eta_x'(s) + \gamma_x(s)\eta_x^2(s)$$

"Phase space invariant"

$$\epsilon_x = \frac{C_q\gamma^2}{J_x}\frac{\oint \frac{\mathcal{H}_x(s)}{|\rho_x|^3}ds}{\oint \frac{1}{\rho_x^2}ds}$$

Equilibrium betatron emittance

$\mathcal{I}_{5_x}$, $\mathcal{I}_3$, $\mathcal{I}_2$, $\mathcal{I}_{4_x}$ — Radiation integrals

$$\sigma_\delta^2 = \frac{C_q\gamma^2}{J_s}\frac{\oint \frac{1}{|\rho_x|^3}ds}{\oint \frac{1}{\rho_x^2}ds}$$

Equilibrium energy spread

$$J_x = 1 - \frac{\oint \frac{\eta_x(s)}{\rho_x^3}(1+2k\rho_x^2)ds}{\oint \frac{1}{\rho_x^2}ds}\ , J_y = 1\ , J_s = 4 - J_x - J_y$$

Damping partition numbers

Reducing the emittance

- Vertical emittance $\epsilon_y \approx 0.01\epsilon_x$ due to coupling

- Horizontal emittance depends on the **energy,** the **bending angle** and the **damping partition number**

$$\epsilon_x \propto \frac{\gamma^2 \theta^3}{J_x}$$

Increase number of dipoles, e.g. from **Double Bend** to **Triple Bend** structure. Difficult due to space constraints

Decrease the energy is not an attractive option for the ESRF (ID's are optimized for 6GeV)

Increase the damping partition number is mostly used for matching and not for emittance minimisation.

Vary field along bending magnet to increase radiation damping, i.e. Double Variable Bend structure

Phase advance for minimum effective emittance cell

- **General rule**: Provided that dispersion is not zero, there is a **unique** phase advance for a straight section with mirror symmetry in the center
- Given the initial (final) optics functions $\beta_0,\ \alpha_0,\ \eta_0,\ \eta_0'$ the phase advance for such a line is

$$\tan(\mu) = \frac{2\eta_0(\beta_0\eta_0' + \alpha_0\eta_0)}{(\beta_0\eta_0' + (\alpha_0 - 1)\eta_0)(\beta_0\eta_0' + (\alpha_0 + 1)\eta_0)}$$

- Applying the result to an arbitrary double bend cell, we obtain

$$\mu_{cell} = \mu_{cell}(\beta_0, \alpha_0, \eta_0, \eta_0', l_d, \theta, \tilde{\theta})$$

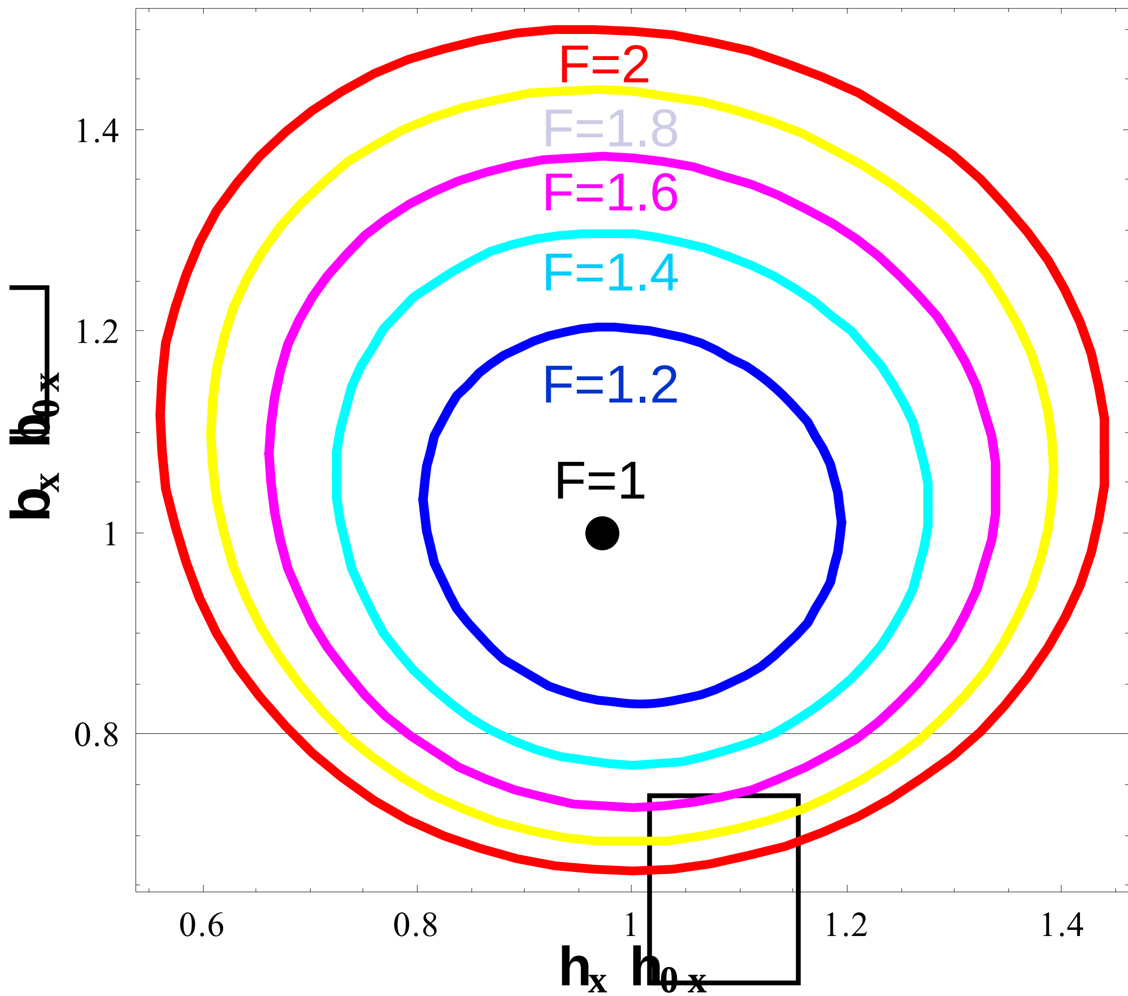
a function depending **only** on the initial optics functions and the dipole !!!

- The horizontal phase advance for reaching the absolute minimum effective emittance at the ESRF storage ring is **293**$^{\circ}$

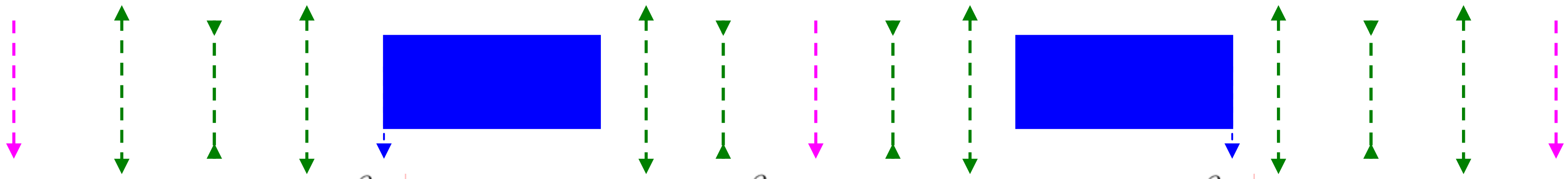
- By detuning the initial beta and dispersion we obtain curves of equal effective emittance ratio

$$F = \frac{\epsilon_{x_{eff}}}{\epsilon_{x_{eff_{min}}}}$$

- Possibility to have a 4-parametric plot for all optics functions
- Note that by detuning the optics functions, the phase advance also changes (**lower** for **higher** F values)



Constraints for general double bend cells



β_{ID1}, $\alpha_{ID1} = 0$, $\eta_{ID1} = \sqrt{\beta_{ID1}\,\mathcal{H}}$, $\eta'_{ID1} = 0$

β_0, α_0, η_0, η'_0

β_{SP}, $\alpha_{SP} = 0$, $\eta_{SP} = \sqrt{\beta_{SP}\,\mathcal{H}}$, $\eta'_{SP} = 0$

β_0, α_0, η_0, η'_0

β_{ID2}, $\alpha_{ID2} = 0$, $\eta_{ID2} = \sqrt{\beta_{ID2}\,\mathcal{H}}$, $\eta'_{ID2} = 0$

- Consider a general double bend with the ideal effective emittance (drifts are parameters)
- In the **straight section** between the ID and the dipole entrance, there are **three constraints**, thus at least **three quadrupoles** are needed
- In the **"achromat",** there are **two constraints**, thus at least **two quadrupoles** are needed (one and a half for a symmetric cell)
- Note that there is **no control** in the vertical plane
- The vertical phase advance is also **fixed**!!!!
- Expressions for the quadrupole gradients can be obtained, parameterized with the drift lengths, the initial optics functions and the beta on the IDs
- All the optics functions are thus uniquely determined for both planes and can be minimized (the gradients as well) by varying the drifts
- The **chromaticities** are also **uniquely defined**

High phase advance implications

Strong quadrupoles

Big phase advance
μ_x

High Chromaticity
$$\xi_{x,y}(s) \propto K(s)\beta_{x,y}(s)$$

Small dispersion
$$\eta(s) \approx \sqrt{\frac{a_c R}{\nu_x}}\sqrt{\beta_x(s)}$$

Strong sextupoles
$$\xi_{x,y}(s) \propto S(s)\eta(s)\beta_{x,y}(s)$$

Small dynamic aperture

Limited injection aperture

Limited lifetime