Magnet Design

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20th June – 1st July 2005

Parameters

- Field needed to bend a particle and force it into a trajectory

$$\vec{F} = q(\vec{v} \times \vec{B}) = m\vec{a} \longrightarrow \begin{array}{c} qvB = mv^2 / R \\ BR = mv / q \end{array}$$

BR = Bρ = p/0.2998 for an electron or a proton when p [GeV]

- Particle species and energy define the accelerator
- The first decision in the design of a circular accelerator is the trade-off between tunnel circumference and magnetic field.
  - Saturated iron magnets. B < 2T
  - > 2T Superconducting technology
  - Emittance of the beam and optics define the quadrupole fields. Strength usually not as stringent.
- Field should be linear to the 10^{-4} level. Transfer function needs to be constant.

Magnetic materials

Remember: $\mathbf{M} = \chi \mathbf{H}$, $\mathbf{B} = \mu \mathbf{H} = \mu_0(\mathbf{H}+\mathbf{M})$

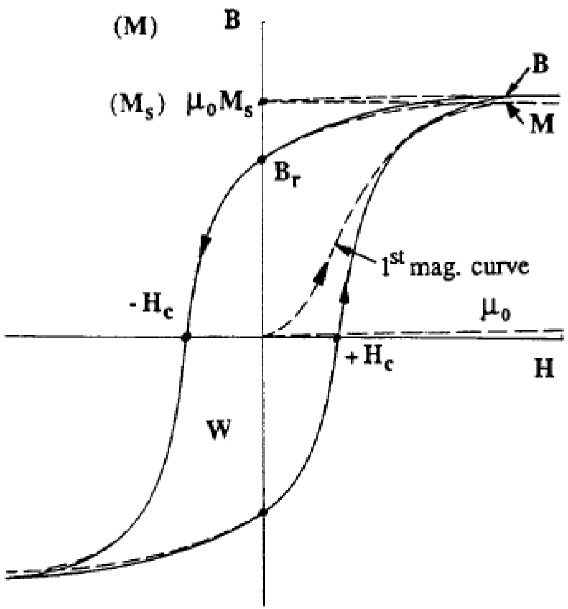
H excitation field, **B** Magnetic flux, Induction, **M** Magnetization Permeability μ, susceptibility χ

Most materials present diamagnetism or paramagnetism. In both cases there is not hysteresis and the magnetization disappears when the external excitation field is zero

Special materials show ferromagnetism. Minuscule magnetic domains are present in the material and align in the presence of a magnetic field.

Saturation is reached asymptotically at 0 ºK with no thermal disorder when all the domains are aligned.

	$\mu_0 M_{sat}$ [Tesla @ 0°K]	Curie Temp [°C]
Iron	2.18	770
Nickel	0.64	358
Cobalt	1.81	1120



Magnetic potential

The Maxwell laws for magneto-statics:

$$\nabla . B = 0;$$

$$\nabla \times H = j.$$

In the absence of currents j=0 we can express B as the gradient of a scalar potential

$$B = \nabla \Phi \quad \text{and}$$

$$\nabla^2 \Phi = 0$$

We get the Laplace equation which has a solution in cylindrical coordinates of the form:

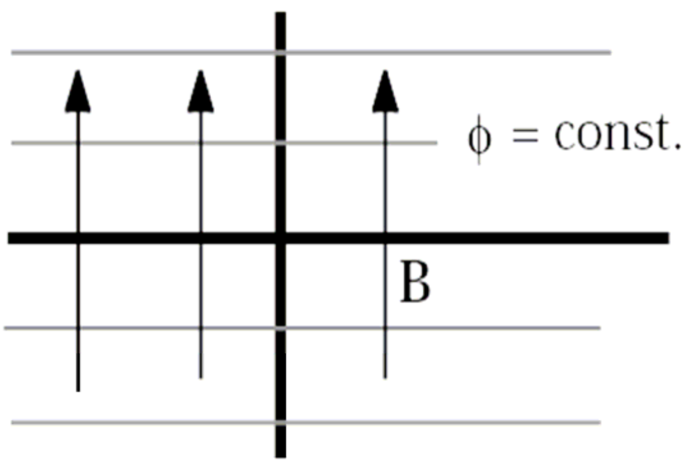
$$\Phi = \sum_{n} \left(A_n r^n \cos(n\theta) + B_n r^n \sin(n\theta) \right)$$

where A_n and B_n correspond to pure skew and normal components

Ideal pole shape

- In iron dominated magnets the field is determined by the shape of the steel poles (lines of constant scalar potential).
- Conductor coils are wounded around the poles to create the magnetic field.
- Due to boundary conditions, pole surfaces are equi-potential line

<u>Dipole (n = 1)</u>



$$\Phi(r,\theta) = A_1 r\cos\theta + B_1 r\sin\theta$$

$$\Phi(x,y) = A_1 x + B_1 y$$

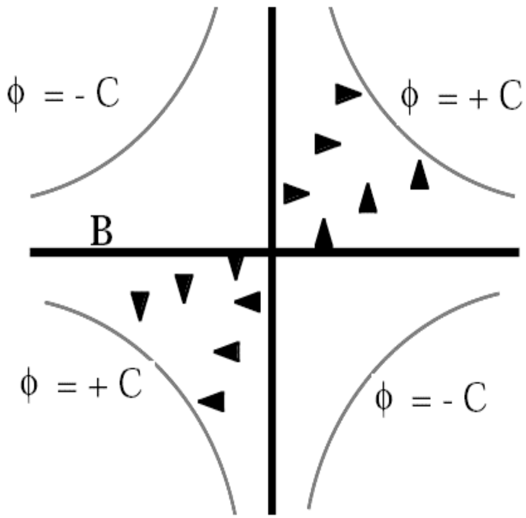
$$\mathrm{B}_x = A_1$$

$$\mathrm{B}_y = B_1$$

$$\text{Pole: } y = \pm g/2$$

Ideal pole shape

Quadrupole (n = 2)



$$\Phi(r,\theta) = A_2 r^2 \cos 2\theta + B_2 r^2 \sin 2\theta$$

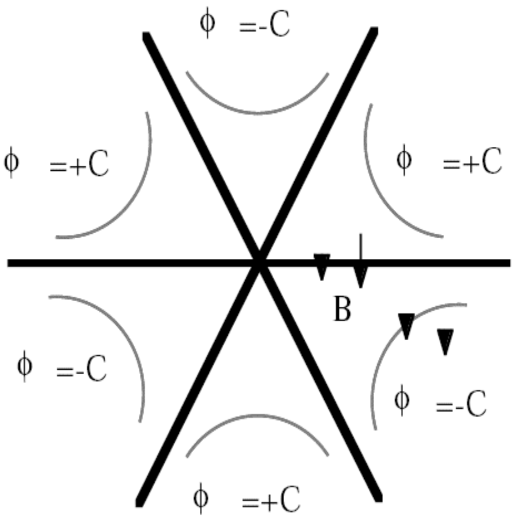
$$\Phi(x,y) = A_2(x^2 - y^2) + 2B_2 xy$$

$$\mathrm{B}_x = 2(A_2 x + B_2 y)$$

$$\mathrm{B}_y = 2(-A_2 y + B_2 x)$$

$$\text{Pole}: xy = \pm R^2/2$$

Sextupole (n = 3)



$$\Phi(r,\theta) = A_3 r^3 \cos 3\theta + B_3 r^3 \sin 3\theta$$

$$\Phi(x,y) = A_3(x^3 - 3y^2 x) + B_3(3yx^2 - y^3)$$

$$\mathrm{B}_x = 3A_3(x^2 - y^2) + 6B_3 xy$$

$$\mathrm{B}_y = -6A_3 xy + 3B_3(x^2 - y^2)$$

$$\text{Pole}: 3x^2 y - y = \pm R^3$$

Symmetries

- Dipole:

$$\Phi(\theta) = -\Phi(2\pi - \theta)$$
$$\Phi(\theta) = \Phi(\pi - \theta)$$

only $\mathbf{B}_n$ with n=1,3,5, etc.

- Quadrupole:

$$\begin{cases} \Phi(\theta) = -\Phi(\pi - \theta) \\ \Phi(\theta) = -\Phi(2\pi - \theta) \end{cases}$$
$$\Phi(\theta) = \Phi(\pi/2 - \theta)$$

only $\mathbf{B}_n$ with n=2,6,10,etc.

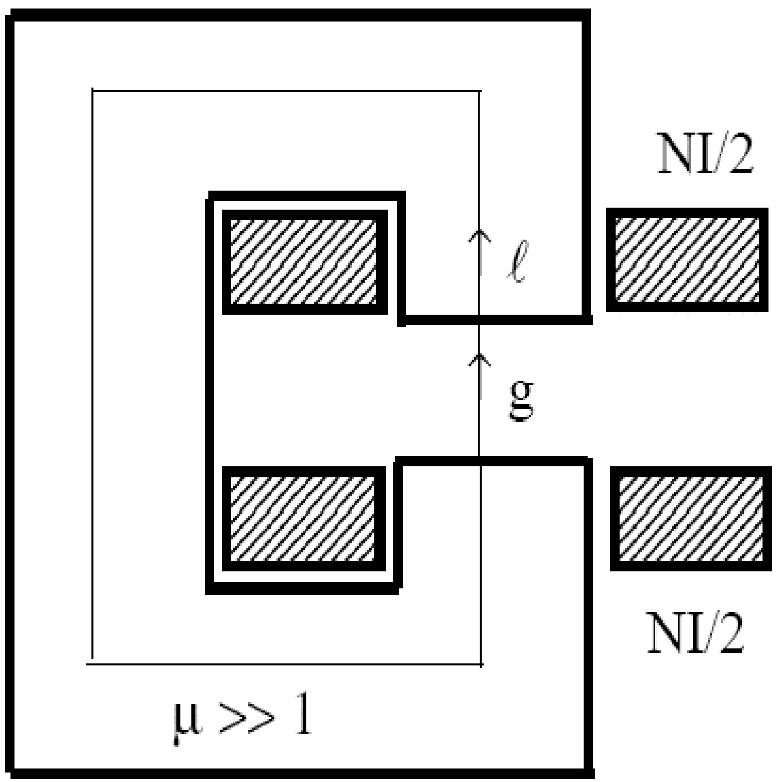
- Sextupole:

$$\begin{cases} \Phi(\theta) = -\Phi(2\pi/3 - \theta) \\ \Phi(\theta) = -\Phi(4\pi/3 - \theta) \\ \Phi(\theta) = -\Phi(2\pi - \theta) \end{cases}$$
$$\Phi(\theta) = \Phi(\pi/3 - \theta)$$

- only $\mathbf{B}_n$ with n=3,9,15,etc.

- In practice poles have a finite width
- Impose the same symmetries to the pole ends to constrain the harmonics
- Longitudinally, the magnet end will generate non linear fringe fields ➔ shims
- Sharp edges will cause saturation and the field becomes non linear ➔ chamfered ends
- Numerical methods applied to define the shape and machined with high precision.

Dipole current



B is constant around the loop l and in the gap

According to Ampere's law

$$\oint \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{A}$$

$$\mathrm{H}_{iron}\, l + \mathrm{H}_{gap}\, g = NI$$

$$\frac{1}{\mu_o \mu_r} \mathrm{B}_{iron}\, l + \frac{1}{\mu_0} \mathrm{B} g = NI$$

with $\mu_r \gg 1$

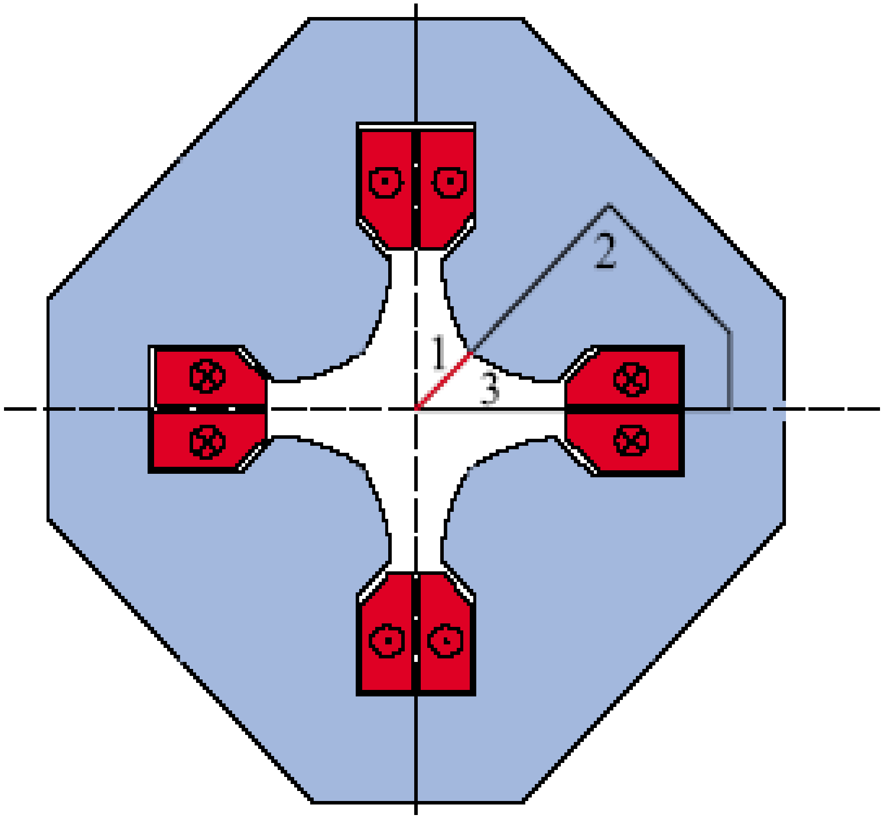
$$\boxed{\mathrm{B} = \frac{\mu_0 NI}{g}}$$

The number of conductors and current density can be optimized

Limited by the iron saturation

$$\mathrm{B} \leq 2\mathrm{T}$$

Quadrupole current



B is perpendicular to ds along path 3

In path 1

Again using Ampere's law

$$\oint \vec{H} \cdot d\vec{s} = \int_1 H_1 \cdot ds + \int_3 H_3 \cdot ds = NI$$

The gradicnt G in a quadrupole is

$$B_x = Gy; \quad B_y = Gx$$

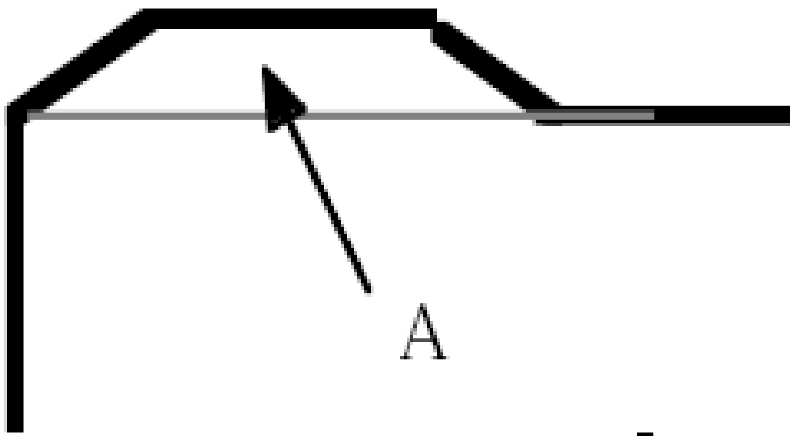
$$\mathrm{H}_2 = \frac{G}{\mu_0}\sqrt{x^2 + y^2} = \frac{G}{\mu_0} r$$

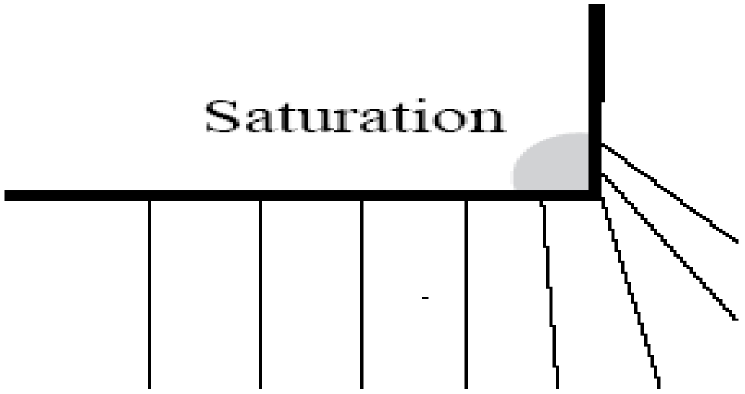
and we get

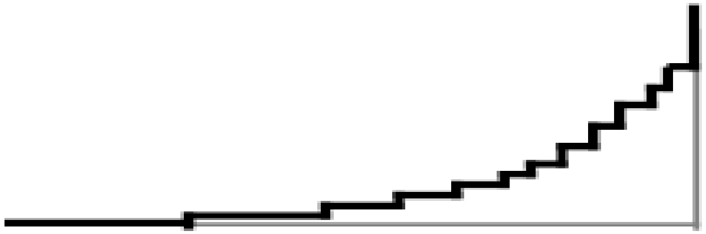
$$\boxed{G = \frac{2\mu_0 NI}{r_0^2}}$$

Practical problems

- The finite pole width creates non linear errors.
- To compensate we add small steps at the outer ends of each of the pole called **shims**.
- Saturation at the corners at the longitudinal end also creates high order multipoles
- **Chamfering** is done using numerical calculations
- Eddy currents oppose the change of magnetic field. They create losses
- Eddy currents in the coil are minimized by **transposing** the conductors in the coil.
- Eddy currents in the yoke are avoided using **laminations**



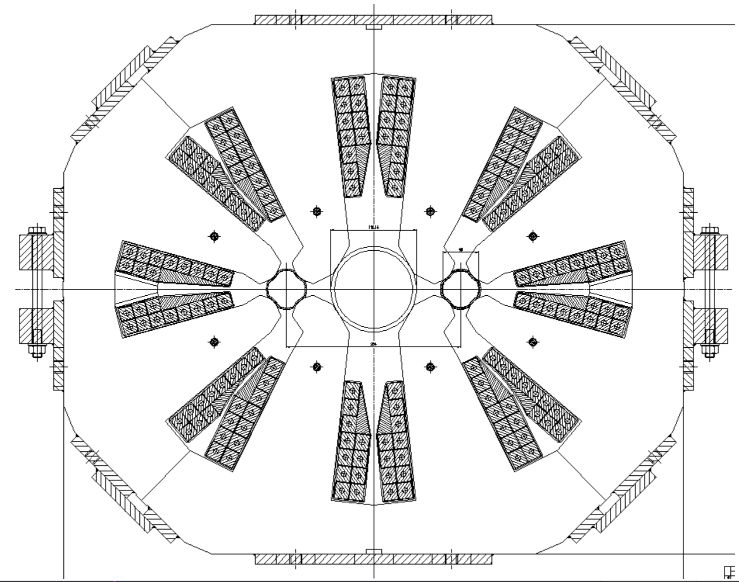


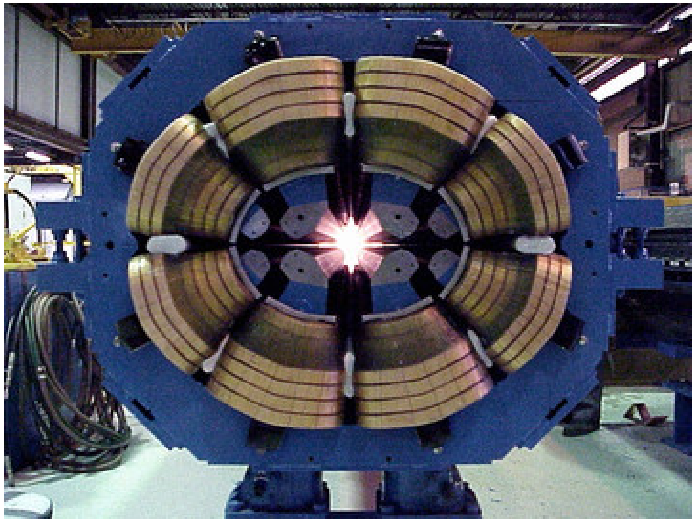


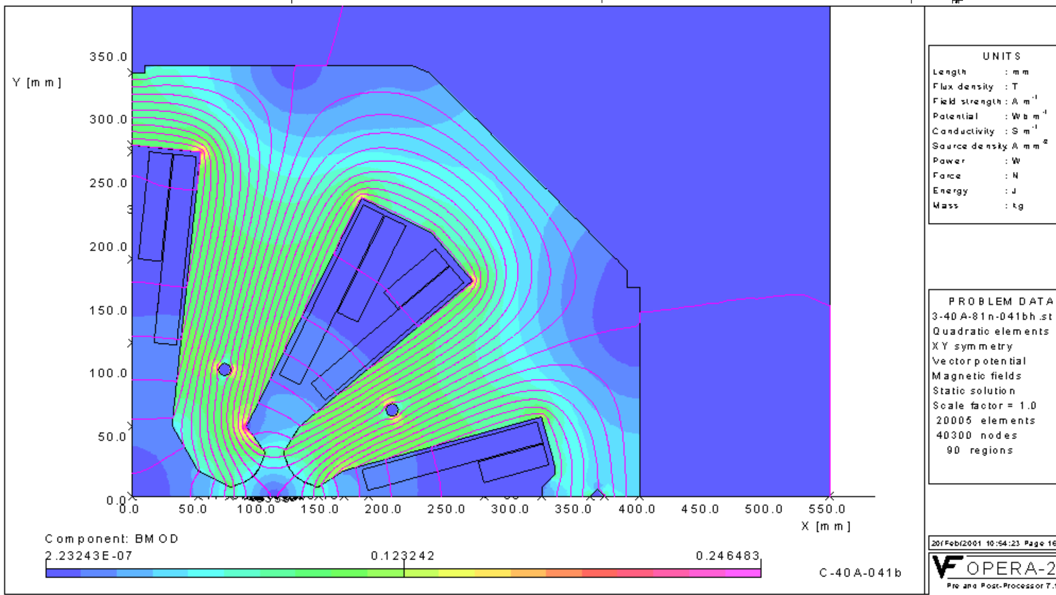
Example: MQWA quadrupole

- Twin aperture quadrupole for LHC built in collaboration with TRIUMF by Alstom, CANADA.
- Pole geometry is identical in the nose
- Disposition of coils around the poles is non symmetric.
- Field calculations require numerical methods done with Opera.

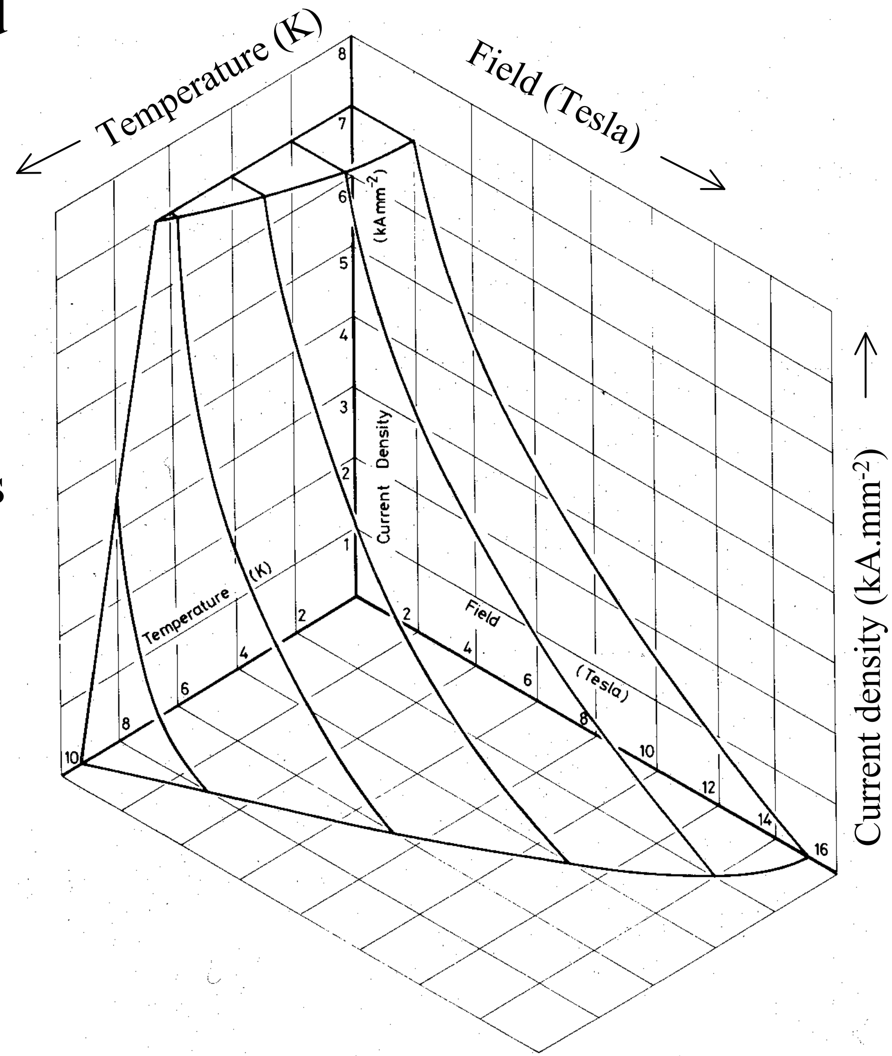
Eva Boter SL-Note-2002-010-MS







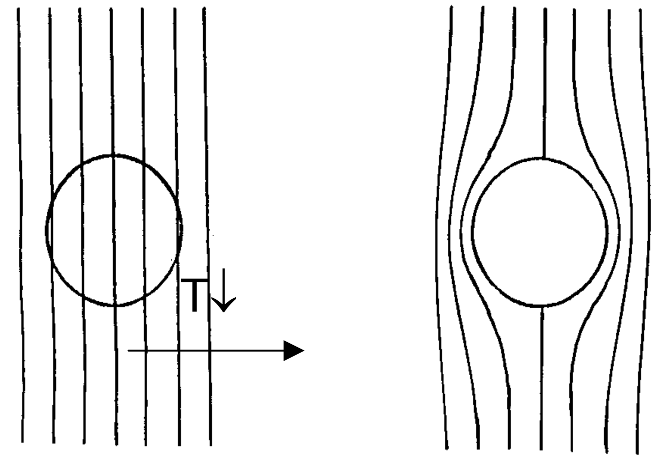
- Superconductivity was discovered in 1911 by H. Kamerlingh Onnes
- The temperature at which the transition takes place is called the critical temperature Tc
- Phase diagram (NbTi). The critical surface.
- Under the surface the material has no resistance
- Type I and Type II superconductors

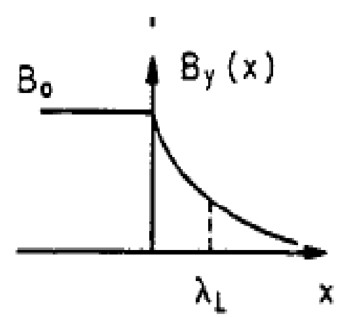


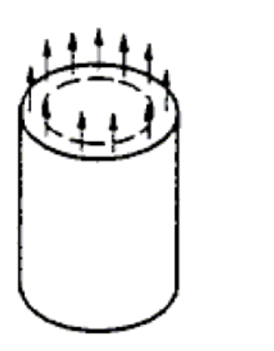
Meissner effect in Type I superconductors

- In 1933 W. Meissner and R.Ochsenfeld discovered what is called the Meissner effect
- The magnetic field is expelled from the sample when it becomes superconducting.
- In strong fields the material goes to normal state.
- Meissner effect is not total, the magnetic field actually penetrates a small distance *l* the London Penetration Depth.
- Current is limited to a small surface ~20-50 nm

  typically for NbTi if $T = 10$ K $Bc = 0.24$T
- The total current is not large enough to create a strong magnetic field.
- Type I superconductors are not suitable for magnet coils







Type II superconductors

- Type II superconductors have a second critical $B_{c2}(T) > B_{c1}(T)$
- Magnetic field lines penetrate the superconductor in the form of flux tubes
- When B increases more and more flux tubes penetrate the material
- However in the presence of a current,

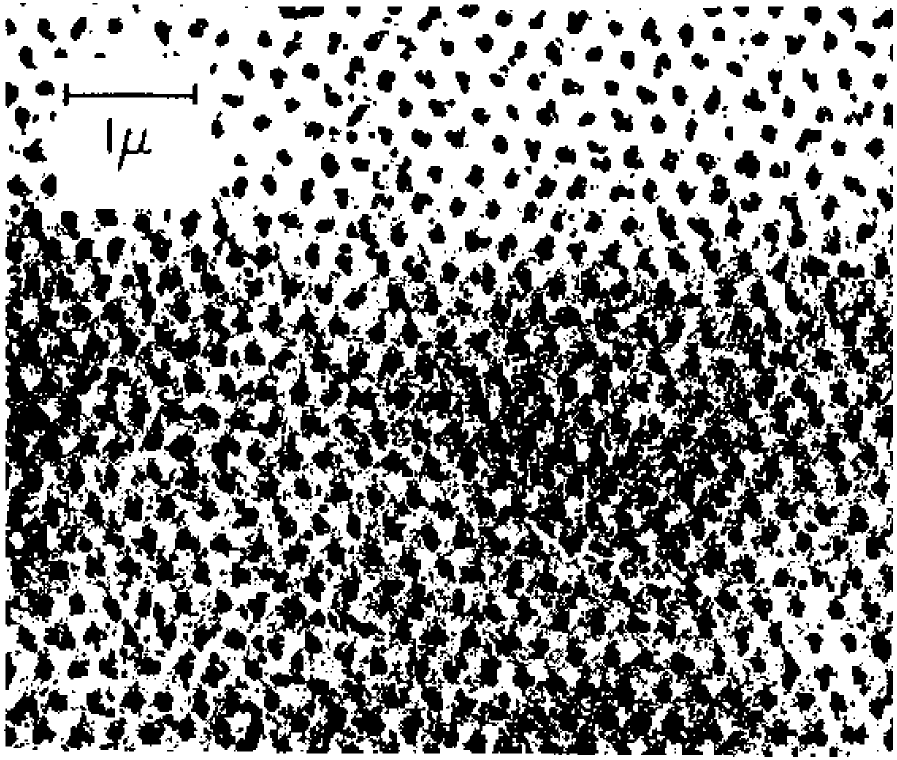
flux tubes move transversely and this motion generates heat

- Introduce flux-pinning centers

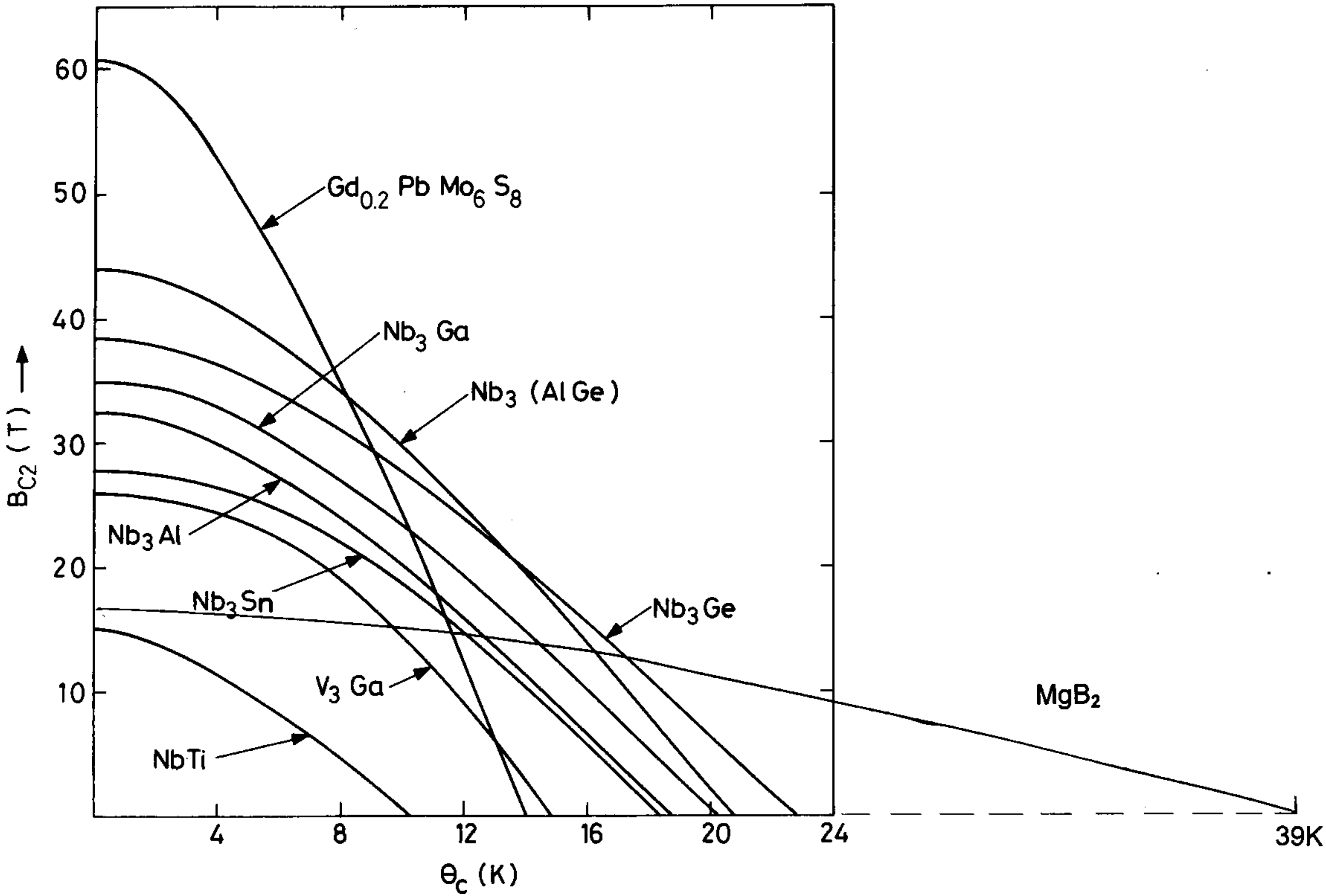
(eg. lattice defects). A very delicate industrial process.

- When the external field decreases

flux is trapped creating hysteresis. (diamagnetic). Needs to warm up the magnet

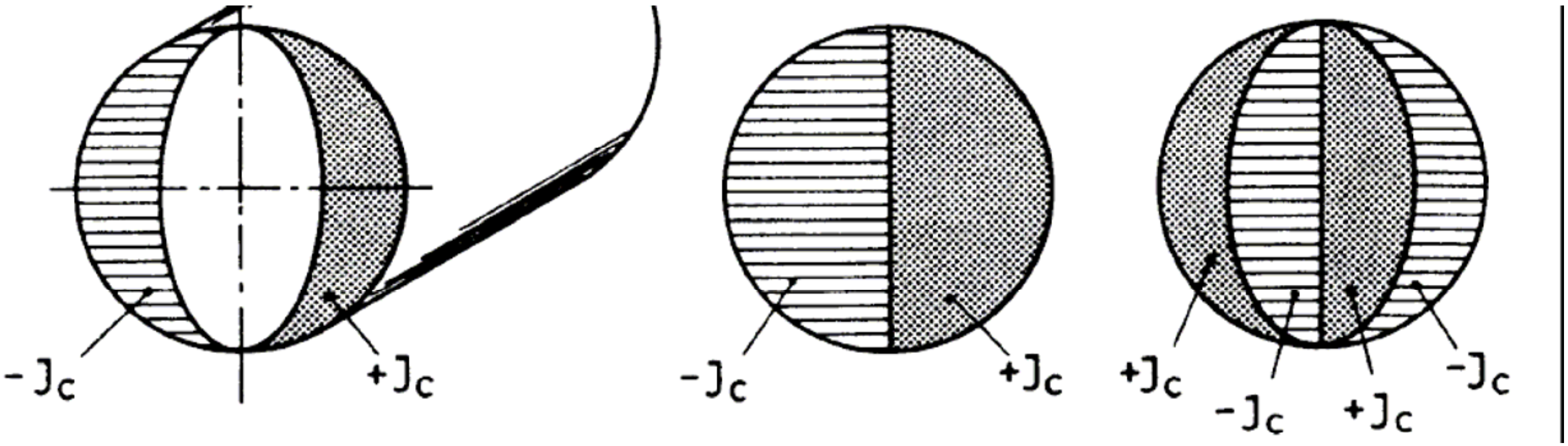


Superconducting materials



Persistent currents

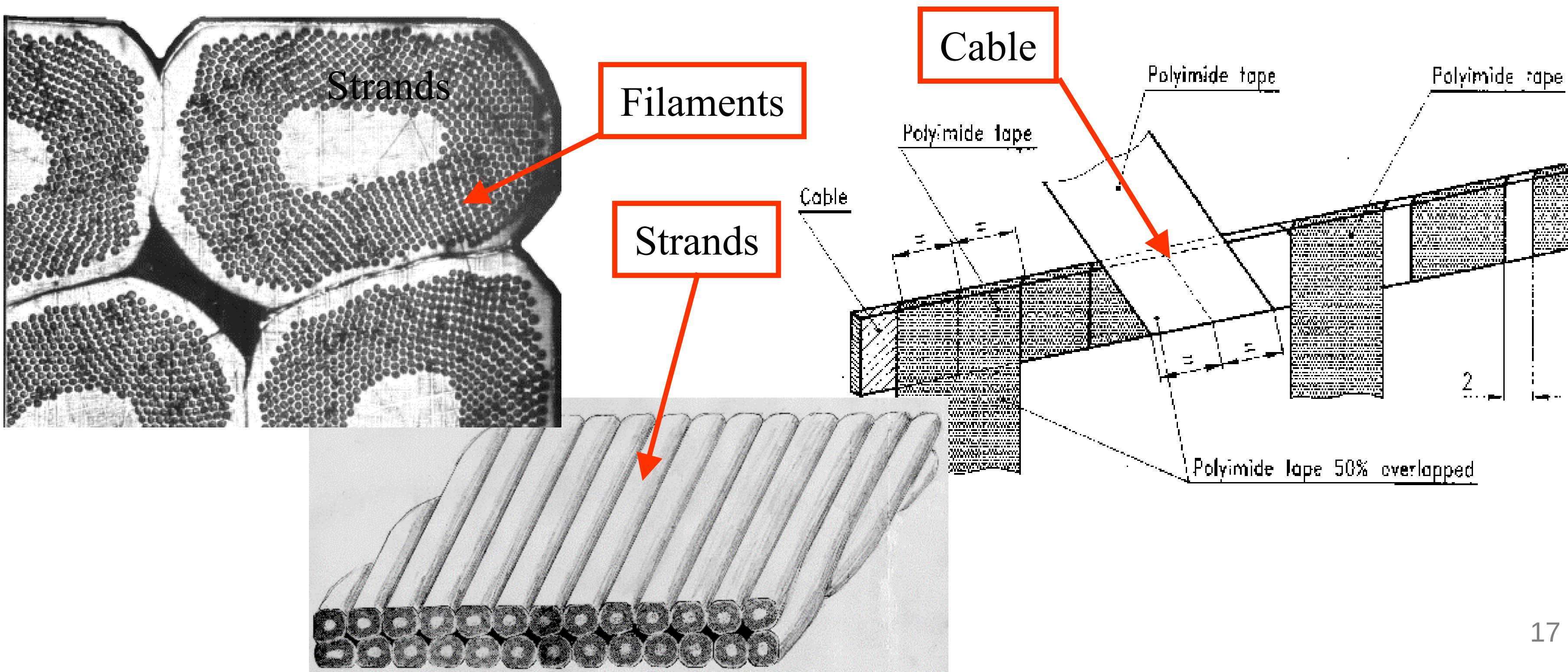
- Currents are produced to counteract a change in magnetic field
- In the copper, eddy currents are minimized by twisting and transposing the strands in the cable
- However in the SC they do not decay



- This persistent currents produce magnetic effects which can be detected outside the cable.
- In order to reduce magnetization effects, all magnet conductors are made with the superconductor divided into fine filaments.
- for NbTi is ~ 50 μm, for accelerator magnets ~ 6-10 μm

Practical superconductors I

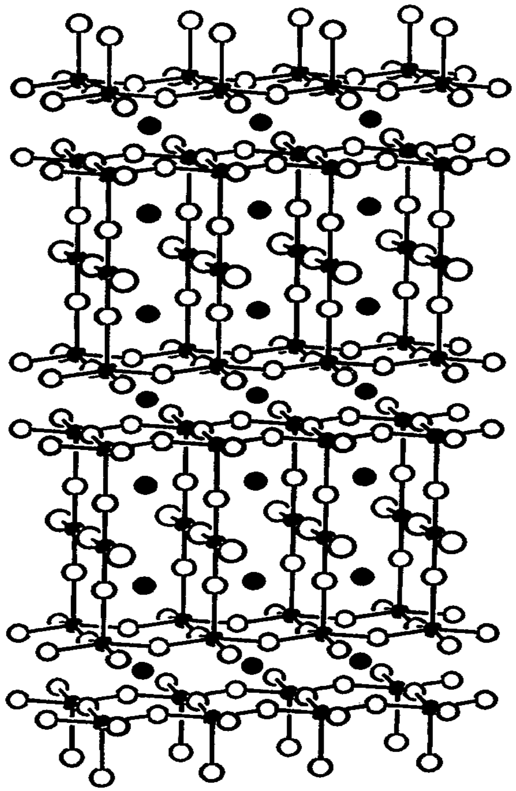
- NbTi has been developed since many years and can be industrially produced in large quantities
- Nb_3Sn is a promising candidate but it is very brittle what makes winding difficult.
- Rutherford cable is the most common used cable in accelerators



Practical superconductors II

- Due to their crystalline structure HT superconductors are highly anisotropic and only conduct in a tiny surface
- Conductors are made out

of a superposition of HTS tape

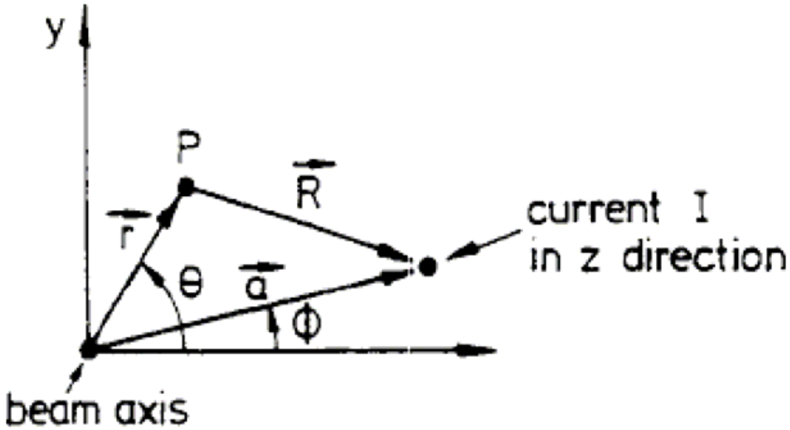


- Used in LHC to manufacture current leads

- Other high temperature superconductors

as MgB_2 made the object of intensive research.

Biot-Savart law



Magnetic potential in the point P created by an infinite line current I

$$A_z(r,\theta) = -\frac{\mu_0 I}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r}{a}\right)^n \cos[n(\phi-\theta)]$$

$$B_\theta = -\frac{\partial A_z}{\partial r} = -\frac{\mu_0 I}{2\pi a}\sum_{n=1}^{\infty}\left(\frac{r}{a}\right)^{n-1} \cos[n(\phi-\theta)]$$

$$B_r = -\frac{1}{r}\frac{\partial A_r}{\partial \theta} = -\frac{\mu_0 I}{2\pi a}\sum_{n=1}^{\infty}\left(\frac{r}{a}\right)^{n-1} \sin[n(\phi-\theta)]$$

A single line current produces only transverse field but multipole fields of any order.

Distribution of current

Assume a distribution of current with the following shape

$$I(\phi) = I_0 \cos(m\phi)$$

Introducing the potential from a current line

$$A_z(r,\theta) = \frac{\mu_0 I_0}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{a}\right)^n \int_0^{2\pi} \cos(m\phi) \cos[n(\theta - \phi)] d\phi$$

Using

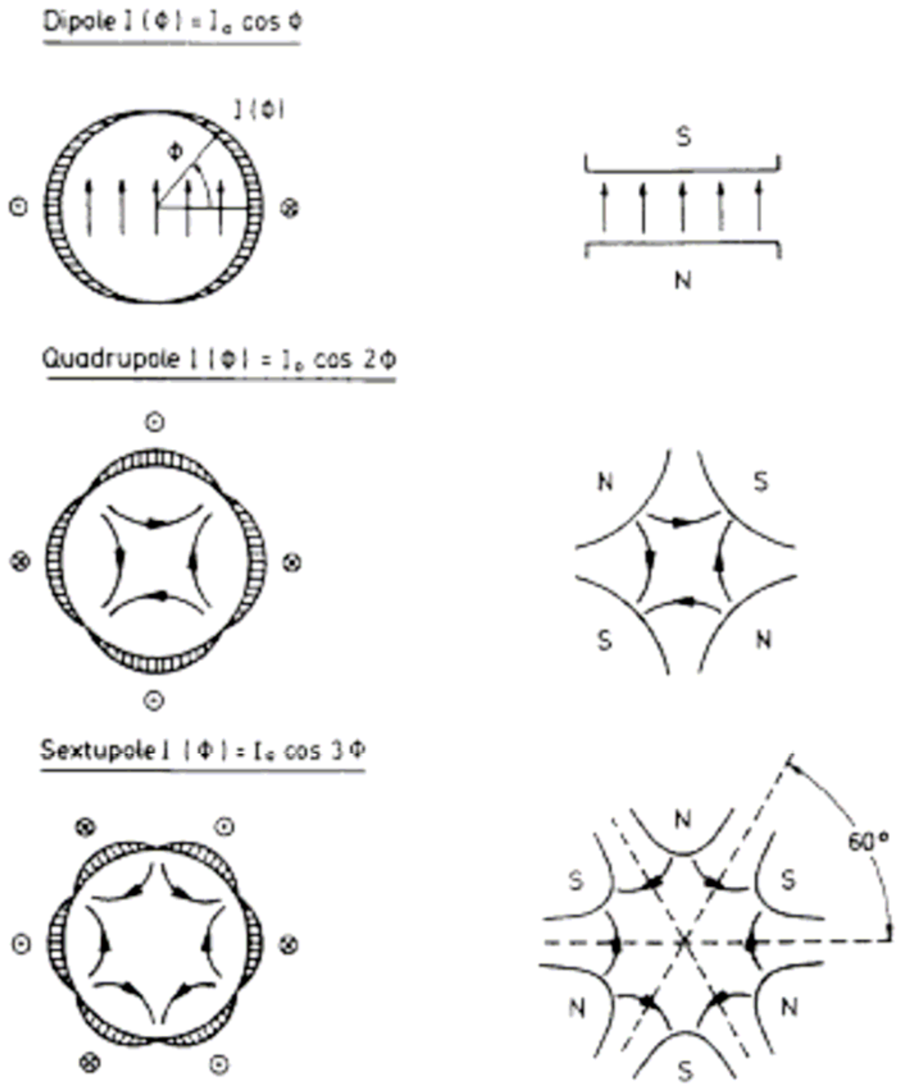
$$\cos[n(\theta-\phi)] = \cos(n\theta)\cos(n\phi) + \sin(n\theta)\sin(n\phi)$$

And using the ortogonality of the trigonometric functions we obtain

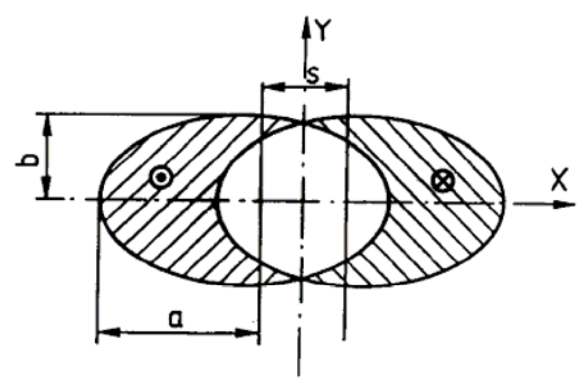
$$A_z(r,\theta) = \frac{\mu_0 I_0}{2} \frac{1}{m} \left(\frac{r}{a}\right)^m \cos(m\theta)$$

<u>A pure multipole of order m!</u>

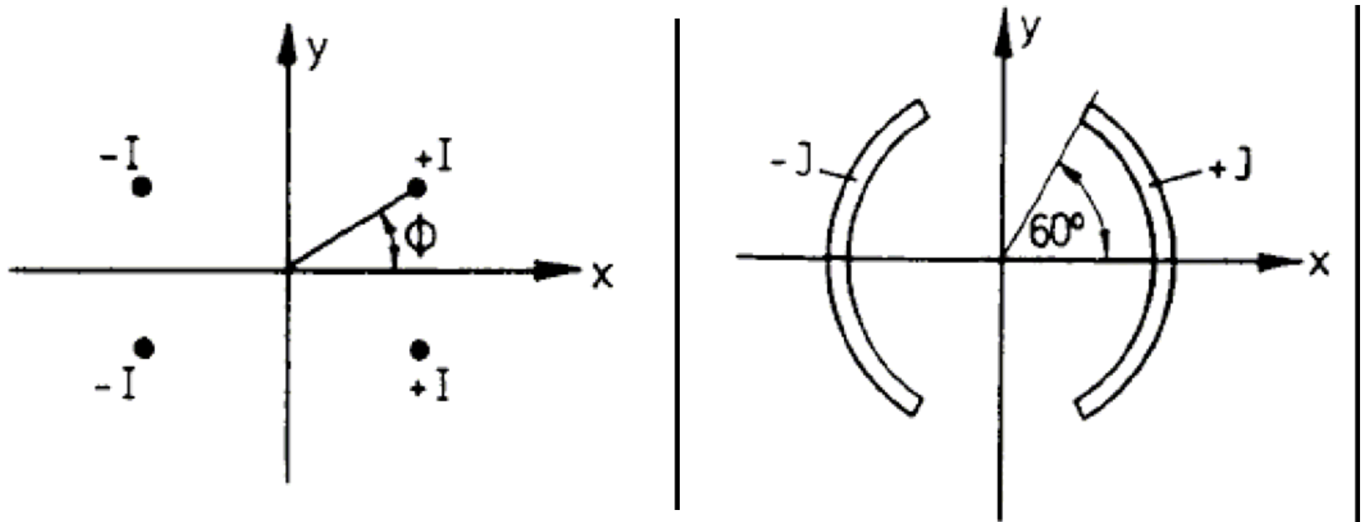
cosϕ magnets



- The yoke is now far away from the magnet center.
- The field quality is determined by the current density
- By using cos(mϕ) current distributions we can generate pure dipoles, quadrupoles and sextupoles
- Intercepting ellipses also create a uniform field



Coil with dipole symmetry



- For any line current +I, it will exist three other currents –I at $\phi+\pi/2$ and $\phi+\pi$ and I at $-\phi$

$$A_z(r,\theta)=\frac{2\mu_0 I}{\pi}\sum_{n=1,3,5}\frac{1}{n}\left(\frac{r}{a}\right)^n\cos(n\phi_1)\cos(n\theta)$$

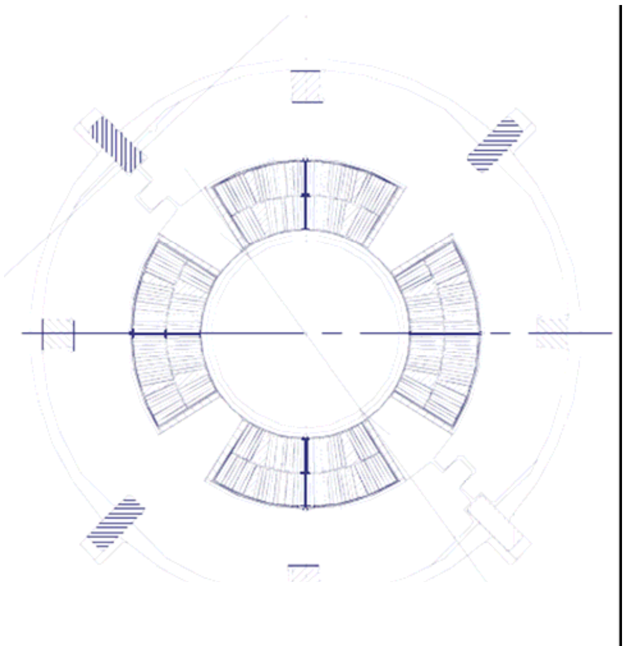
- A coil with dipole symmetry produces only odd normal multipoles

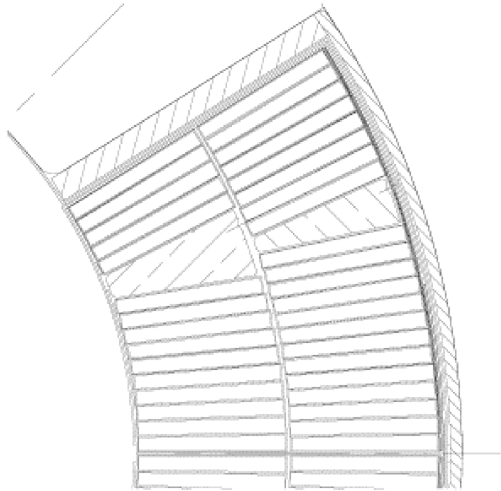
$$B_n=\frac{2\mu_0 J}{\pi}\Delta a\frac{1}{n}\left(\frac{r}{a}\right)^{n-1}\cos(n\phi_1)$$

- By taking ϕ_1=60, the sextupole term n=3 vanishes!!

Practical geometry

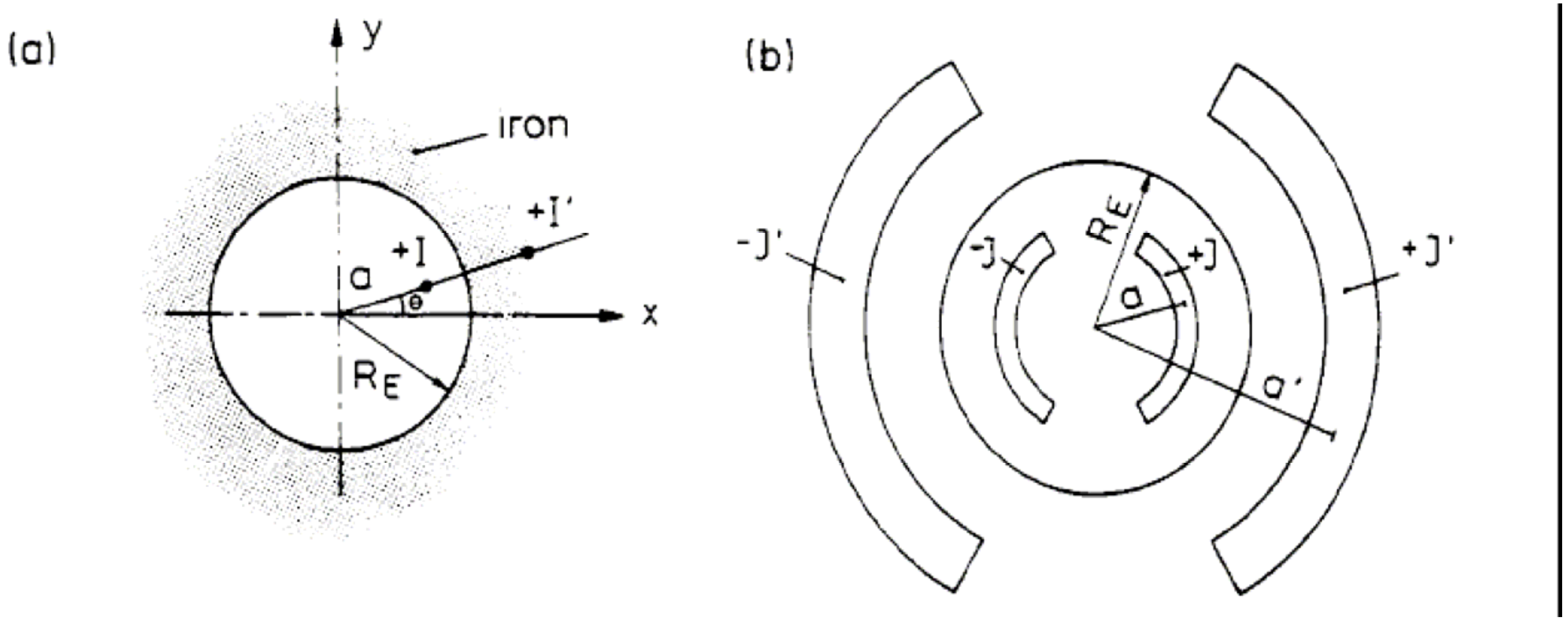
- In practice, coils are winded from real cable with an approximate rectangular section.
- Several shells or layers approximate the cos(mϕ) distribution.
- The symmetries seen before are respected
- Errors are reduced by introducing wedges
- Numerical methods are required to achieve the required main field and minimize the other harmonics.





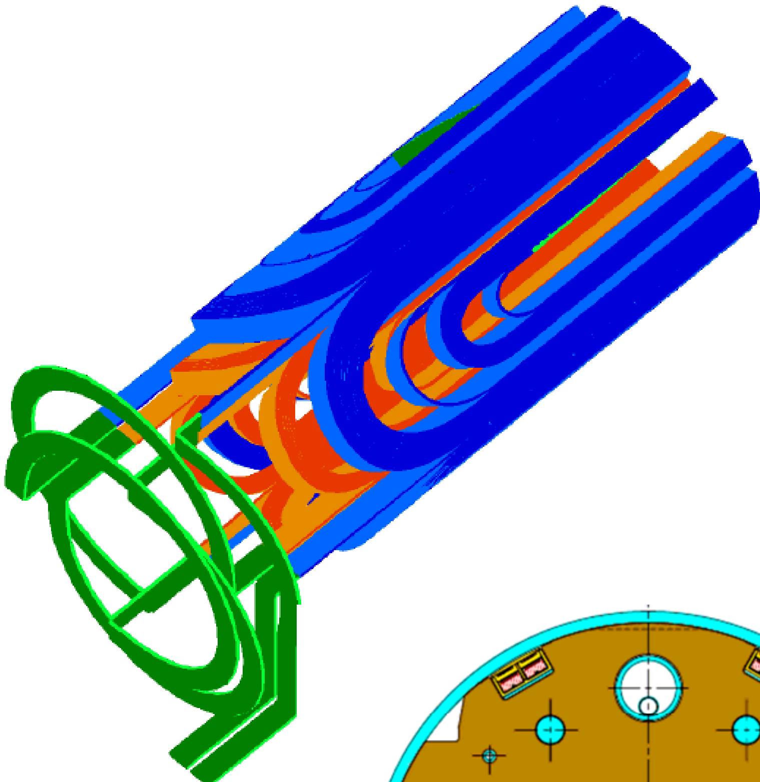
Influence of the iron yoke

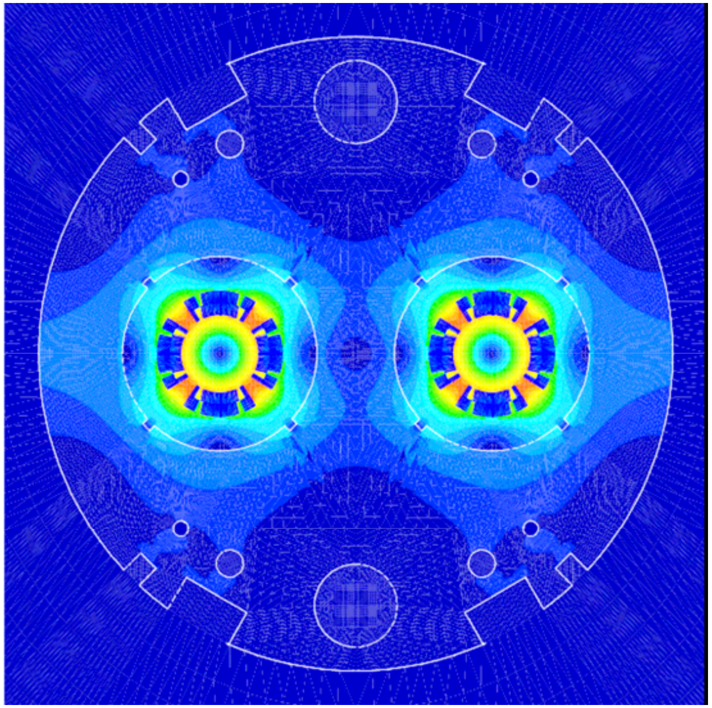
- Magnetic field lines return inside the yoke
- The magnitude of the main magnetic field in the center of the magnet increases 10-20%
- Image methods are used to calculate the total effect.
- Does not work at saturation where transfer function is not longer linear
- Numerical methods need to be used in that case

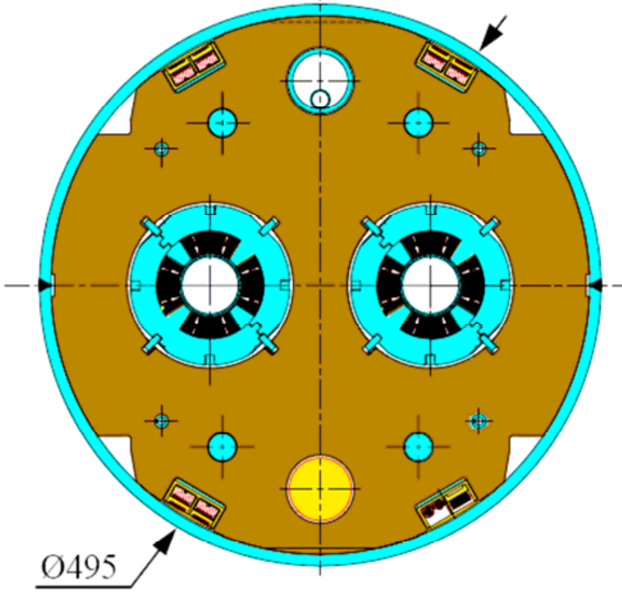


Numerical methods

- Codes like Roxie, Mafia, Superfish etc.. are used in order to calculate position the coils, ends, calculate the harmonics, forces, stored energy etc.

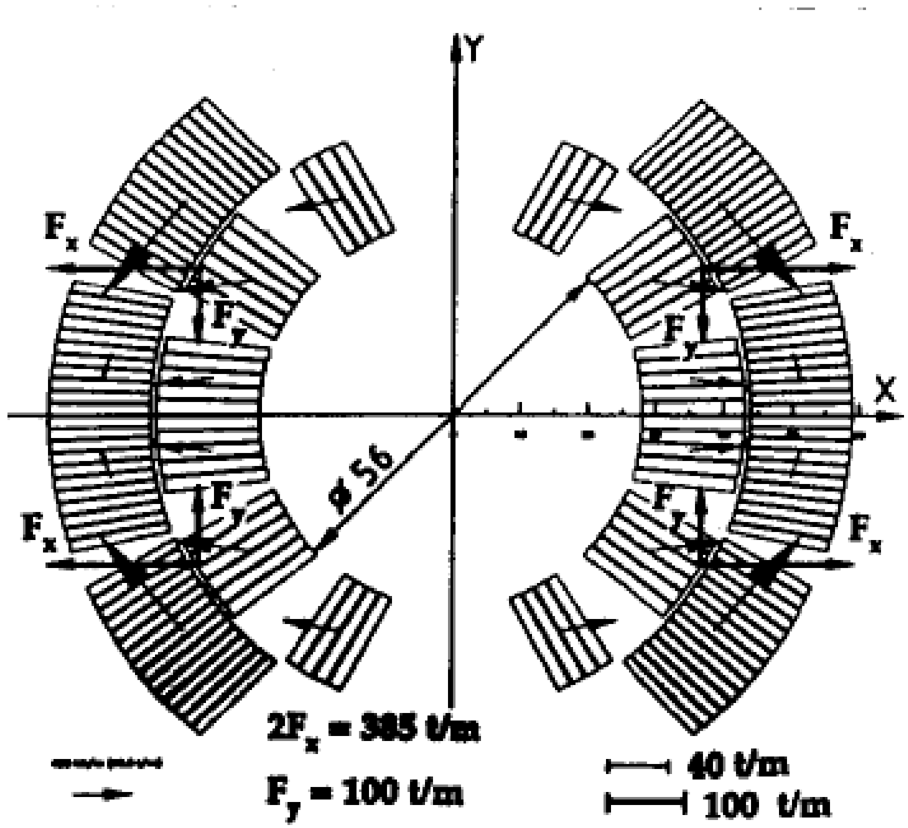






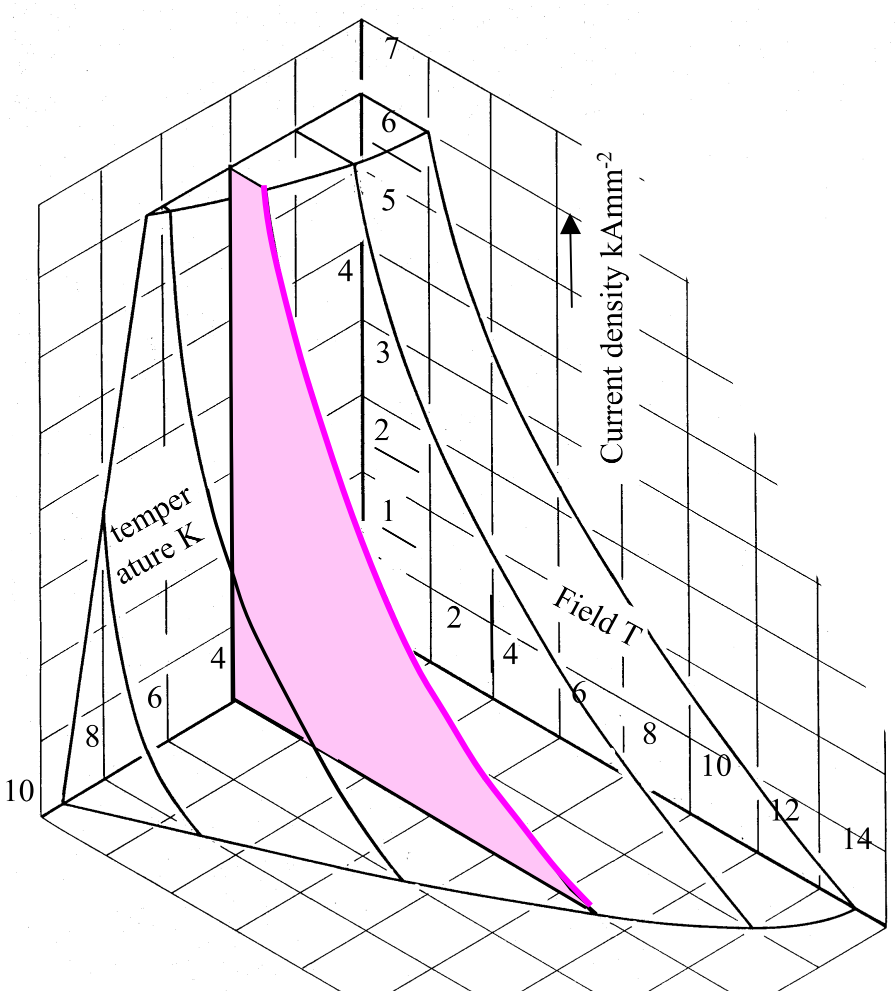
Mechanic constraints

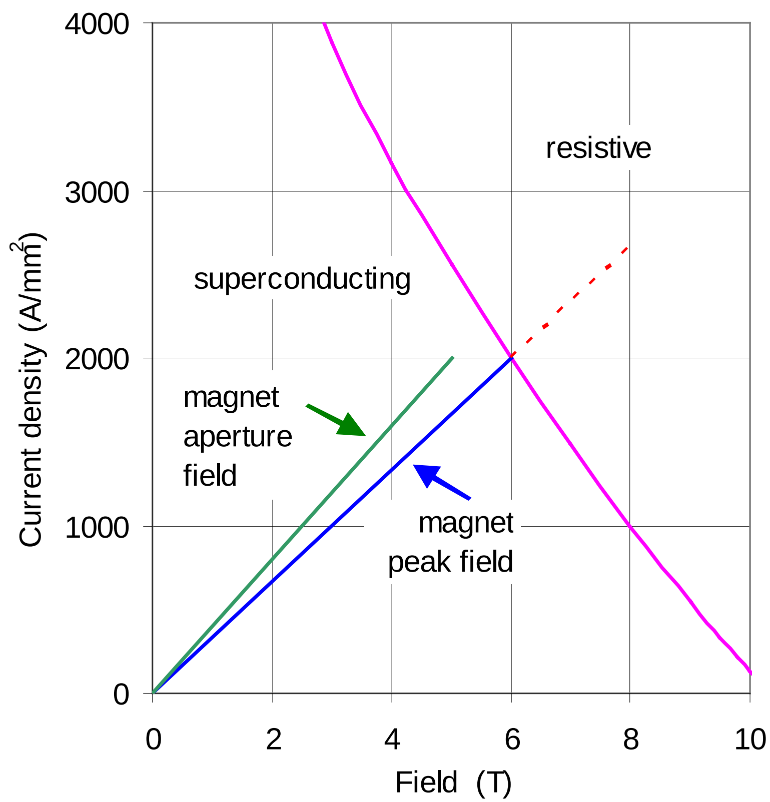
- Lorenz forces act upon the coils when powered. LHC dipole Fx~1.6×106 N/m = 160tons/m
- The different materials from which the magnet is made, have different shrinking coefficients
- Coils may become loose in cryogenic conditions
- Large pre-stress of the order of MPa is is needed to avoid small movements during operation.
- Also harmful multipoles when coils are displaced.
- Coils, shrinking cylinders, welds under pressure, impregnation, pancakes…



Operating loadline

Magnet operates in a load line where field is linear with current density





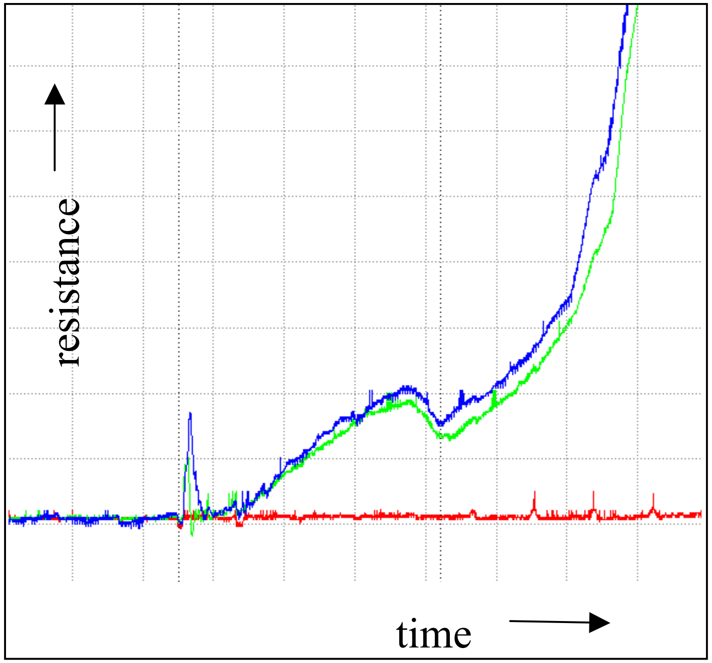
temperature rise may be caused by
- ac losses
- poor joints
- beam energy deposition
-etc, etc

The load line automatically changes

Quench propagation

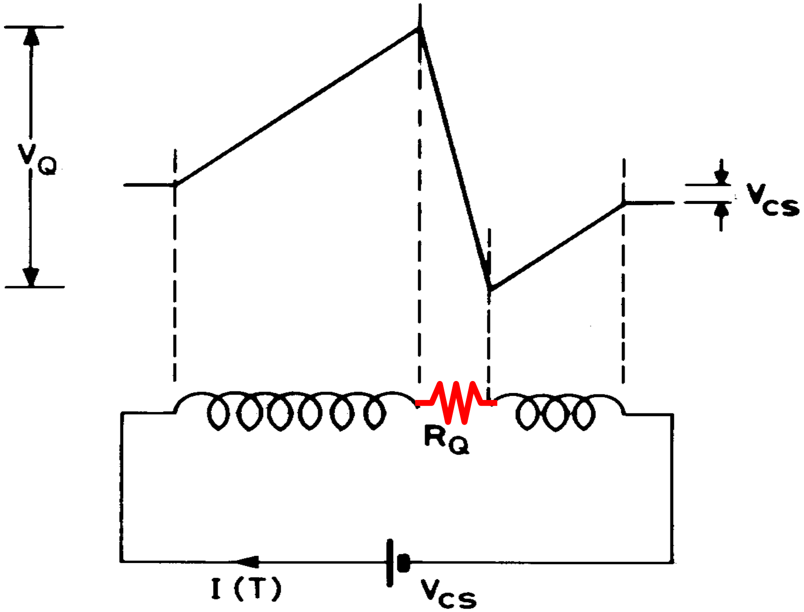
Quench propagation

- A normal conducting zone appears in the SC where heat is generated
- If heat is conducted out of the resistive zone faster than it is generated, the zone will shrink - vice versa it will grow. MPZ minimum propagating zone
internal voltages much greater than terminal voltage



Quench protection:

- detect the quench electronically
- power a heater in good thermal contact with the winding
- this quenches other regions of the magnet, effectively forcing the normal zone to grow more rapidly (higher resistance, shorter decay time ,lower temperature rise at the hot spot)



Magnetic Measurements

- To preserve dynamic aperture of the beam during hours of operation
- To accept large beams within a large linear region
- In both normal and superconducting magnets, magnetic field needs to be linear to the 10^{-4} level!!

<u>Ferromagnetic</u>

- Pole ends
- Iron saturation

<u>Superconducting</u>

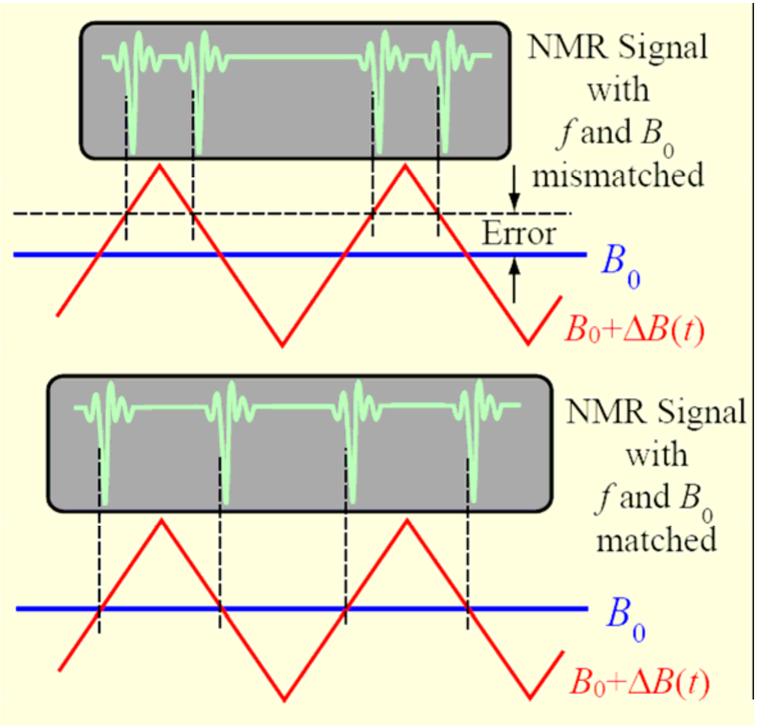
- Coil ends
- Persistent currents
- Decay and Snap-back
- Saturation of the yoke

- Magnetic field and harmonics need to be measured with a 10^{-6} precision

Nuclear Magnetic Resonance

NMR Nuclear Magnetic resonance

From A. Jain CERN Academic training



- A particle with a net spin placed in a magnetic field of strength B has two energy levels.
- The energy gap between levels is proportional to the field and the gyromagnetic ratio, γ of the particle.
- A resonant absorption of RF energy occurs at a frequency corresponding to energy gap.

$$\Delta E = \gamma h B$$

Hall effect

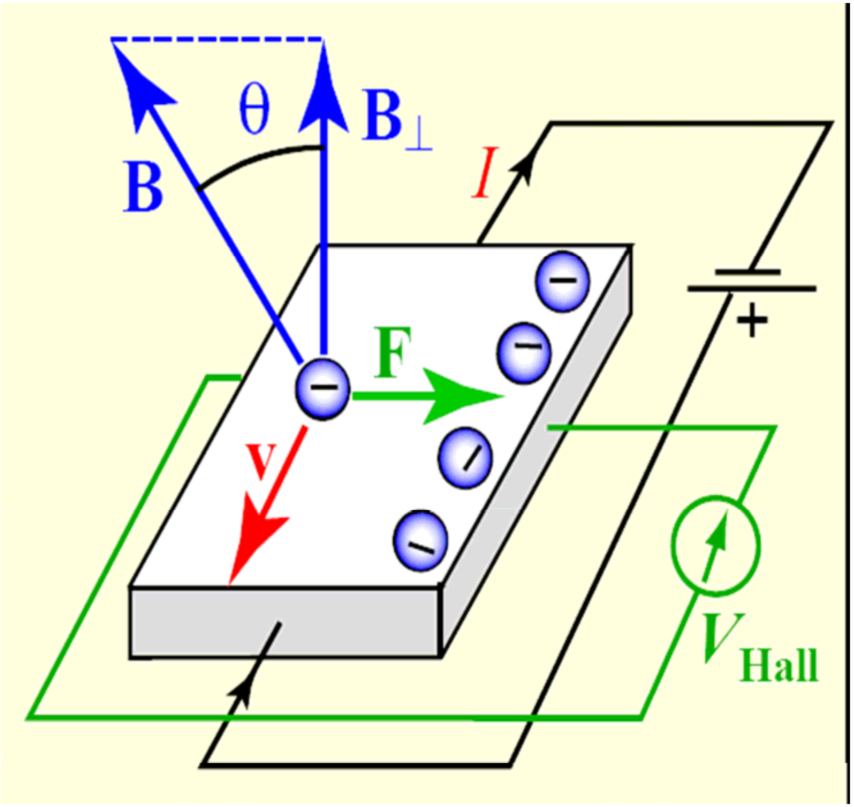
Hall probe

- Charge carriers experience a **Lorentz force** in the presence of a magnetic field.
- This produces a steady state voltage in a direction perpendicular to the current and field.

$$V_{Hall} = G R_H I \, \mathrm{B} \cos\theta$$

G = geometric factor

R_H = Hall coefficient



However, the response of the Hall probe to the field direction is very complex!!

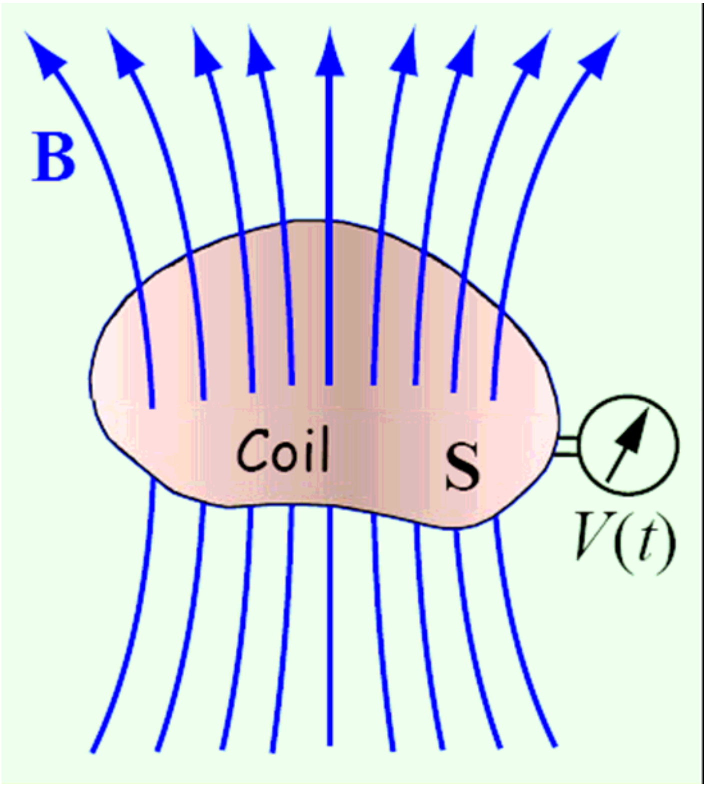
Rotating coils

- A change of flux in the coil induces a voltage.

$$\int_{t_{start}}^{t_{end}} V(t) = -\Phi_{end} - \Phi_{start}$$

- By using a rotating coil we make sure that

$$\Phi_{end} = \Phi_{start}$$

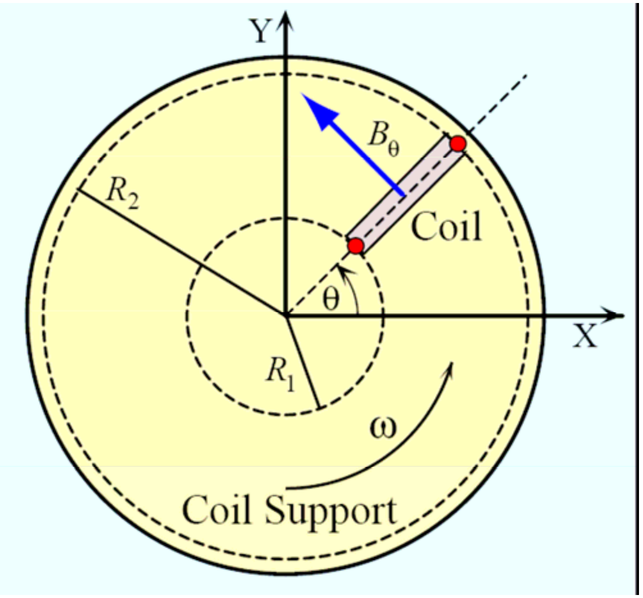


Rotating coil

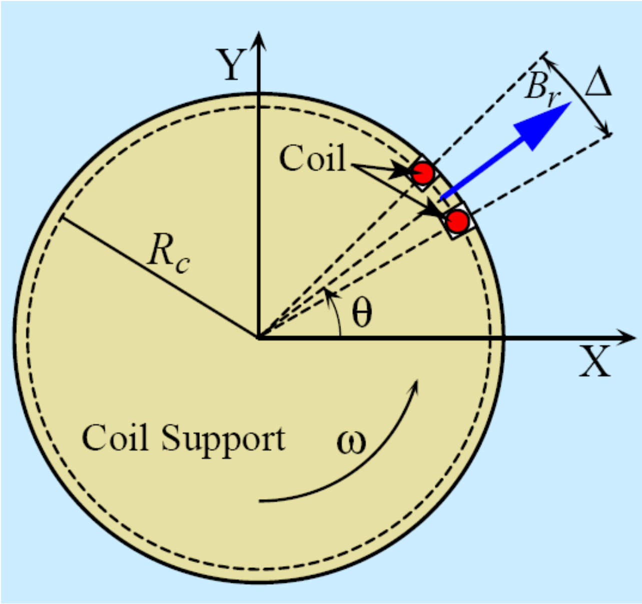
- Remember

$$B_r(r,\theta)=\sum_{n=1}^{\infty}\left(\frac{r}{R_{ref}}\right)^n\left[B_n\sin(n\theta)+A_n\cos(n\theta)\right]$$

$$B_\theta(r,\theta)=\sum_{n=1}^{\infty}\left(\frac{r}{R_{ref}}\right)^n\left[B_n\cos(n\theta)-A_n\sin(n\theta)\right]$$



Radial coil sensitive to the azimuthal field



Tangential coil sensitive to the radial field

Signal induced in a radial coil

- The signal produced in a radial coil will be

$$V(t) = \sum_{n=1}^{\infty} NLR_{ref}\,\omega \left[\left(\frac{R_2}{R_{ref}} \right)^n - \left(\frac{R_1}{R_{ref}} \right)^n \right] \left[B_n \sin(n\omega t + n\delta) + A_n \cos(n\omega t + n\delta) \right]$$

N = number of turns

L = length

δ = angle at $t = 0$

ω = angular velocity

$\phi = \omega t + \delta$

The periodic variation of coil voltage is also described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, the geometric parameters of the coil, and angular velocity.

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