Magnet Design

Y. Papaphilippou, N. Catalan Lasheras

USPAS, Cornell University, Ithaca, NY
20th June – 1st July 2005
Parameters

- Field needed to bend a particle and force it into a trajectory

\[ F = q(\mathbf{v} \times \mathbf{B}) = ma \quad \rightarrow \quad qvB = \frac{mv^2}{R} \]

\[ BR = B\rho = \frac{p}{0.2998} \text{ for an electron or a proton when } p \text{ [GeV]} \]

- Particle species and energy define the accelerator

- The first decision in the design of a circular accelerator is the trade-off between tunnel circumference and magnetic field.
  - Saturated iron magnets. \( B < 2T \)
  - \( > 2T \) Superconducting technology
  - Emittance of the beam and optics define the quadrupole fields. Strength usually not as stringent.

- Field should be linear to the \( 10^{-4} \) level. Transfer function needs to be constant.
Magnetic materials

Remember: \( M = \chi H, \ B = \mu H = \mu_0(H+M) \)

- **\( H \)** excitation field, \( \mathbf{B} \) Magnetic flux, Induction, \( \mathbf{M} \) Magnetization Permeability \( \mu \), susceptibility \( \chi \)

Most materials present diamagnetism or paramagnetism. In both cases there is not hysteresis and the magnetization disappears when the external excitation field is zero.

Special materials show ferromagnetism. Minuscule magnetic domains are present in the material and align in the presence of a magnetic field.

Saturation is reached asymptotically at 0 \(^\circ\)K with no thermal disorder when all the domains are aligned.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \mu_0 M_{\text{sat}} ) [Tesla @ 0(^\circ)K]</th>
<th>Curie Temp [(^\circ)C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>2.18</td>
<td>770</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.64</td>
<td>358</td>
</tr>
<tr>
<td>Cobalt</td>
<td>1.81</td>
<td>1120</td>
</tr>
</tbody>
</table>
The Maxwell laws for magneto-statics:

\[ \nabla \cdot B = 0; \]
\[ \nabla \times H = j. \]

In the absence of currents \( j = 0 \) we can express \( B \) as the gradient of a scalar potential

\[ B = \nabla \Phi \quad \text{and} \]
\[ \nabla^2 \Phi = 0 \]

We get the Laplace equation which has a solution in cylindrical coordinates of the form:

\[ \Phi = \sum_n \left( A_n r^n \cos(n\theta) + B_n r^n \sin(n\theta) \right) \]

where \( A_n \) and \( B_n \) correspond to pure skew and normal components
In iron dominated magnets the field is determined by the shape of the steel poles (lines of constant scalar potential).

Conductor coils are wounded around the poles to create the magnetic field.

Due to boundary conditions, pole surfaces are equi-potential lines.

**Dipole (n = 1)**

\[ \Phi(r, \theta) = A_1 r \cos \theta + B_1 r \sin \theta \]

\[ \Phi(x, y) = A_1 x + B_1 y \]

\[ B_x = A_1 \]

\[ B_y = B_1 \]

Pole: \( y = \pm g / 2 \)
Ideal pole shape

Quadrupole (n = 2)

\[ \Phi(r, \theta) = A_2 r^2 \cos 2\theta + B_2 r^2 \sin 2\theta \]
\[ \Phi(x, y) = A_2 (x^2 - y^2) + 2B_2 xy \]
\[ B_x = 2(A_2 x + B_2 y) \]
\[ B_y = 2(-A_2 y + B_2 x) \]
Pole: \( xy = \pm R^2 / 2 \)

Sextupole (n = 3)

\[ \Phi(r, \theta) = A_3 r^3 \cos 3\theta + B_3 r^3 \sin 3\theta \]
\[ \Phi(x, y) = A_3 (x^3 - 3y^2 x) + B_3 (3yx^2 - y^3) \]
\[ B_x = 3A_3 (x^2 - y^2) + 6B_3 xy \]
\[ B_y = -6A_3 xy + 3B_3 (x^2 - y^2) \]
Pole: \( 3x^2 y - y = \pm R^3 \)
Symmetries

- **Dipole:**
  \[
  \Phi(\theta) = -\Phi(2\pi - \theta) \\
  \Phi(\theta) = \Phi(\pi - \theta)
  \]
  only \(B_n\) with \(n=1,3,5,\) etc.

- **Quadrupole:**
  \[
  \begin{cases}
  \Phi(\theta) = -\Phi(\pi - \theta) \\
  \Phi(\theta) = -\Phi(2\pi - \theta) \\
  \Phi(\theta) = \Phi(\pi / 2 - \theta)
  \end{cases}
  \]
  only \(B_n\) with \(n=2,6,10,\) etc.

- **Sextupole:**
  \[
  \begin{cases}
  \Phi(\theta) = -\Phi(2\pi / 3 - \theta) \\
  \Phi(\theta) = -\Phi(4\pi / 3 - \theta) \\
  \Phi(\theta) = -\Phi(2\pi - \theta) \\
  \Phi(\theta) = \Phi(\pi / 3 - \theta)
  \end{cases}
  \]
  only \(B_n\) with \(n=3,9,15,\) etc.

- In practice poles have a finite width
- Impose the same symmetries to the pole ends to constrain the harmonics
- Longitudinally, the magnet end will generate non-linear fringe fields \(\rightarrow\) shims
- Sharp edges will cause saturation and the field becomes non-linear \(\rightarrow\) chamfered ends
- Numerical methods applied to define the shape and machined with high precision.
Dipole current

\[ \mathbf{B} \text{ is constant around the loop } l \text{ and in the gap} \]

According to Ampere’s law

\[ \oint \mathbf{H} \cdot d\mathbf{s} = \int \mathbf{J} \cdot d\mathbf{A} \]

\[ \mathbf{H}_{\text{iron}} l + \mathbf{H}_{\text{gap}} g = NI \]

\[ \frac{1}{\mu_0 \mu_r} B_{\text{iron}} l + \frac{1}{\mu_0} B g = NI \]

with \( \mu_r \gg 1 \)

\[ B = \frac{\mu_0 NI}{g} \]

The number of conductors and current density can be optimized

Limited by the iron saturation

\[ B \leq 2T \]
Quadrupole current

\( \mathbf{B} \) is perpendicular to \( ds \) along path 3

In path 1

Again using Ampere’s law

\[
\int \mathbf{H} \cdot d\mathbf{s} = \int_{1} H_{1} \cdot ds + \int_{3} H_{3} \cdot ds = NI
\]

The gradient \( G \) in a quadrupole is

\[
B_{x} = Gy; \quad B_{y} = Gx
\]

\[
H_2 = \frac{G}{\mu_0} \sqrt{x^2 + y^2} = \frac{G}{\mu_0} r
\]

and we get

\[
G = \frac{2\mu_0 NI}{r_0^2}
\]
Practical problems

- The finite pole width creates non-linear errors.
- To compensate we add small steps at the outer ends of each of the pole called shims.
- Saturation at the corners at the longitudinal end also creates high order multipoles.
- Chamfering is done using numerical calculations.
- Eddy currents oppose the change of magnetic field. They create losses.
- Eddy currents in the coil are minimized by transposing the conductors in the coil.
- Eddy currents in the yoke are avoided using laminations.
Example: MQWA quadrupole

- Twin aperture quadrupole for LHC built in collaboration with TRIUMF by Alstom, CANADA.
- Pole geometry is identical in the nose.
- Disposition of coils around the poles is non symmetric.
- Field calculations require numerical methods done with Opera.
Superconductivity was discovered in 1911 by H. Kamerlingh Onnes.

The temperature at which the transition takes place is called the critical temperature $T_c$.

Phase diagram (NbTi). The critical surface.

Under the surface the material has no resistance.

Type I and Type II superconductors.
Meissner effect in Type I superconductors

- In 1933 W. Meissner and R. Ochsenfeld discovered what is called the Meissner effect.
- The magnetic field is expelled from the sample when it becomes superconducting.
- In strong fields the material goes to normal state.
- Meissner effect is not total, the magnetic field actually penetrates a small distance \( l \) the London Penetration Depth.
- Current is limited to a small surface \( \sim 20-50 \) nm
  - typically for NbTi if \( T = 10 \) K \( B_c = 0.24T \)
- The total current is not large enough to create a strong magnetic field.
- Type I superconductors are not suitable for magnet coils.
Type II superconductors

- Type II superconductors have a second critical $B_{c2}(T) > B_{c1}(T)$
- Magnetic field lines penetrate the superconductor in the form of flux tubes
- When $B$ increases more and more flux tubes penetrate the material
- However in the presence of a current, flux tubes move transversely and this motion generates heat
- Introduce flux-pinning centers (e.g. lattice defects). A very delicate industrial process.
- When the external field decreases flux is trapped creating hysteresis. (diamagnetic). Needs to warm up the magnet
Superconducting materials
Persistent currents

- Currents are produced to counteract a change in magnetic field
- In the copper, eddy currents are minimized by twisting and transposing the strands in the cable
- However in the SC they do not decay

- This persistent currents produce magnetic effects which can be detected outside the cable.
- In order to reduce magnetization effects, all magnet conductors are made with the superconductor divided into fine filaments.
- for NbTi is ~ 50 μm, for accelerator magnets ~ 6-10 μm
- NbTi has been developed since many years and can be industrially produced in large quantities.
- Nb3Sn is a promising candidate but it is very brittle what makes winding difficult.
- Rutherford cable is the most common used cable in accelerators.
Due to their crystalline structure HT superconductors are highly anisotropic and only conduct in a tiny surface.

Conductors are made out of a superposition of HTS tape.

Used in LHC to manufacture current leads.

Other high temperature superconductors as MgB$_2$ made the object of intensive research.
Magnetic potential in the point P created by an infinite line current I

\[
A_z(r, \theta) = -\frac{\mu_0 I}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{r}{a} \right)^n \cos[n(\phi - \theta)]
\]

\[
B_\theta = -\frac{\partial A_z}{\partial r} = -\frac{\mu_0 I}{2\pi a} \sum_{n=1}^{\infty} \left( \frac{r}{a} \right)^{n-1} \cos[n(\phi - \theta)]
\]

\[
B_r = -\frac{1}{r} \frac{\partial A_r}{\partial \theta} = -\frac{\mu_0 I}{2\pi a} \sum_{n=1}^{\infty} \left( \frac{r}{a} \right)^{n-1} \sin[n(\phi - \theta)]
\]

A single line current produces only transverse field but multipole fields of any order.
Distribution of current

Assume a distribution of current with the following shape

\[ I(\phi) = I_0 \cos(m\phi) \]

Introducing the potential from a current line

\[ A_z(r, \theta) = \frac{\mu_0 I_0}{2\pi} \sum_{n=1}^{\infty} \left( \frac{r}{a} \right)^n \int_{0}^{\infty} \cos(m\phi) \cos(n(\theta - \phi)) d\phi \]

Using

\[ \cos[n(\theta - \phi)] = \cos(n\theta) \cos(n\phi) + \sin(n\theta) \sin(n\phi) \]

And using the orthogonality of the trigonometric functions we obtain

\[ A_z(r, \theta) = \frac{\mu_0 I_0}{2} \frac{1}{m} \left( \frac{r}{a} \right)^m \cos(m\theta) \]

A pure multipole of order m!
cosφ magnets

- The yoke is now far away from the magnet center.
- The field quality is determined by the current density.
- By using cos(mφ) current distributions we can generate pure dipoles, quadrupoles and sextupoles.
- Intercepting ellipses also create a uniform field.
Coil with dipole symmetry

- For any line current +I, it will exist three other currents –I at φ+π/2 and φ+π and I at -φ

\[ A_z(r, \theta) = \frac{2\mu_0 I}{\pi} \sum_{n=1,3,5} \frac{1}{n} \left( \frac{r}{a} \right)^n \cos(n\phi_1)\cos(n\theta) \]

- A coil with dipole symmetry produces only odd normal multipoles

\[ B_n = \frac{2\mu_0 J}{\pi} \Delta a \frac{1}{n} \left( \frac{r}{a} \right)^{n-1} \cos(n\phi_1) \]

- By taking \( \phi_1 = 60 \), the sextupole term \( n=3 \) vanishes!!
Practical geometry

- In practice, coils are wound from real cable with an approximate rectangular section.
- Several shells or layers approximate the \( \cos(m\phi) \) distribution.
- The symmetries seen before are respected.
- Errors are reduced by introducing wedges.
- Numerical methods are required to achieve the required main field and minimize the other harmonics.
Influence of the iron yoke

- Magnetic field lines return inside the yoke
- The magnitude of the main magnetic field in the center of the magnet increases 10-20%
- Image methods are used to calculate the total effect.
- Does not work at saturation where transfer function is not longer linear
- Numerical methods need to be used in that case
Numerical methods

- Codes like Roxie, Mafia, Superfish etc. are used in order to calculate position the coils, ends, calculate the harmonics, forces, stored energy etc.
Mechanics constraints

- Lorenz forces act upon the coils when powered. LHC dipole $F_x \approx 1.6 \times 10^6$ N/m = 160tons/m
- The different materials from which the magnet is made, have different shrinking coefficients
- Coils may become loose in cryogenic conditions
- Large pre-stress of the order of MPa is needed to avoid small movements during operation.
- Also harmful multipoles when coils are displaced.
- Coils, shrinking cylinders, welds under pressure, impregnation, pancakes...
Magnet operates in a load line where field is linear with current density.

The load line automatically changes.

Temperature rise may be caused by:
- ac losses
- poor joints
- beam energy deposition
- etc, etc

The load line automatically changes.
Quench propagation

Quench propagation
- A normal conducting zone appears in the SC where heat is generated.
- If heat is conducted out of the resistive zone faster than it is generated, the zone will shrink; vice versa it will grow. MPZ minimum propagating zone internal voltages much greater than terminal voltage.

Quench protection:
- detect the quench electronically
- power a heater in good thermal contact with the winding
- this quenches other regions of the magnet, effectively forcing the normal zone to grow more rapidly (higher resistance, shorter decay time, lower temperature rise at the hot spot)
Magnetic Measurements

- To preserve dynamic aperture of the beam during hours of operation
- To accept large beams within a large linear region
- In both normal and superconducting magnets, magnetic field needs to be linear to the $10^{-4}$ level!!

**Ferromagnetic**
- Pole ends
- Iron saturation

**Superconducting**
- Coil ends
- Persistent currents
- Decay and Snap-back
- Saturation of the yoke

- Magnetic field and harmonics need to be measured with a $10^{-6}$ precision
Nuclear Magnetic Resonance

NMR Nuclear Magnetic resonance

- A particle with a net spin placed in a magnetic field of strength $B$ has two energy levels.
- The energy gap between levels is proportional to the field and the gyromagnetic ratio, $\gamma$, of the particle.
- A resonant absorption of RF energy occurs at a frequency corresponding to energy gap.

$$\Delta E = \gamma \hbar B$$

From A. Jain CERN Academic training
Hall effect

Hall probe

- Charge carriers experience a **Lorentz force** in the presence of a magnetic field.
- This produces a steady state voltage in a direction perpendicular to the current and field.

\[ V_{\text{Hall}} = GR_H I B \cos \theta \]

G = geometric factor

\( R_H \) = Hall coefficient

However, the response of the Hall probe to the field direction is very complex!!
**Flux measurements**

**Rotating coils**

- A change of flux in the coil induces a voltage.

\[
\int_{t_{\text{start}}}^{t_{\text{end}}} V(t) = -\Phi_{\text{end}} - \Phi_{\text{start}}
\]

- By using a rotating coil we make sure that

\[
\Phi_{\text{end}} = \Phi_{\text{start}}
\]
Rotating coil

- Remember

\[
B_r(r, \theta) = \sum_{n=1}^{\infty} \left( \frac{r}{R_{ref}} \right)^n \left[ B_n \sin(n\theta) + A_n \cos(n\theta) \right]
\]

\[
B_\theta(r, \theta) = \sum_{n=1}^{\infty} \left( \frac{r}{R_{ref}} \right)^n \left[ B_n \cos(n\theta) - A_n \sin(n\theta) \right]
\]

Radial coil sensitive to the azimuthal field

Tangential coil sensitive to the radial field
The signal produced in a radial coil will be

\[
V(t) = \sum_{n=1}^{\infty} NLR_{ref} \omega \left[ \left( \frac{R_2}{R_{ref}} \right)^n - \left( \frac{R_1}{R_{ref}} \right)^n \right] \left[ B_n \sin(n \omega t + n \delta) + A_n \cos(n \omega t + n \delta) \right]
\]

N = number of turns
L = length
\( \delta \) = angle at \( t = 0 \)
\( \omega \) = angular velocity
\( \phi = \omega t + \delta \)

The periodic variation of coil voltage is also described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, the geometric parameters of the coil, and angular velocity.
“Materials”, J. Billan in “CERN Accelerator School: Magnetic measurement and alignment” CERN 92-05

“Conventional Magnets”, N. Marks in “CERN Accelerator School: 5th General accelerator physics course” CERN 94-01


“Superconductivity”, P. Schmuser, in “CERN Accelerator School: Superconductivity in particle accelerators” CERN 96-03

“1st International Roxie Users Meeting and Workshop”, CERN 99-01

“Magnetic Field Measurements and Mapping Techniques”, A. Jain CERN Academic training program. 2003