Effective emittance reduction using variable field dipoles in electron storage rings

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Lattice upgrade options

- Vertical emittance $\epsilon_y \approx 0.01\epsilon_x$ due to coupling

- Horizontal emittance depends on the **energy,** the **bending angle** and the **damping partition number**

$$\epsilon_x \propto \frac{\gamma^2 \theta^3}{J_x}$$

Increase number of dipoles, e.g. from **Double Bend** to **Triple Bend** structure. Difficult due to space constraints

Decrease the energy is not an attractive option for the ESRF (ID's are optimized for 6GeV)

Vary field along bending magnet to increase radiation damping, i.e. Double Variable Bend structure

Increase the damping partition number is mostly used for matching and not for emittance minimisation.

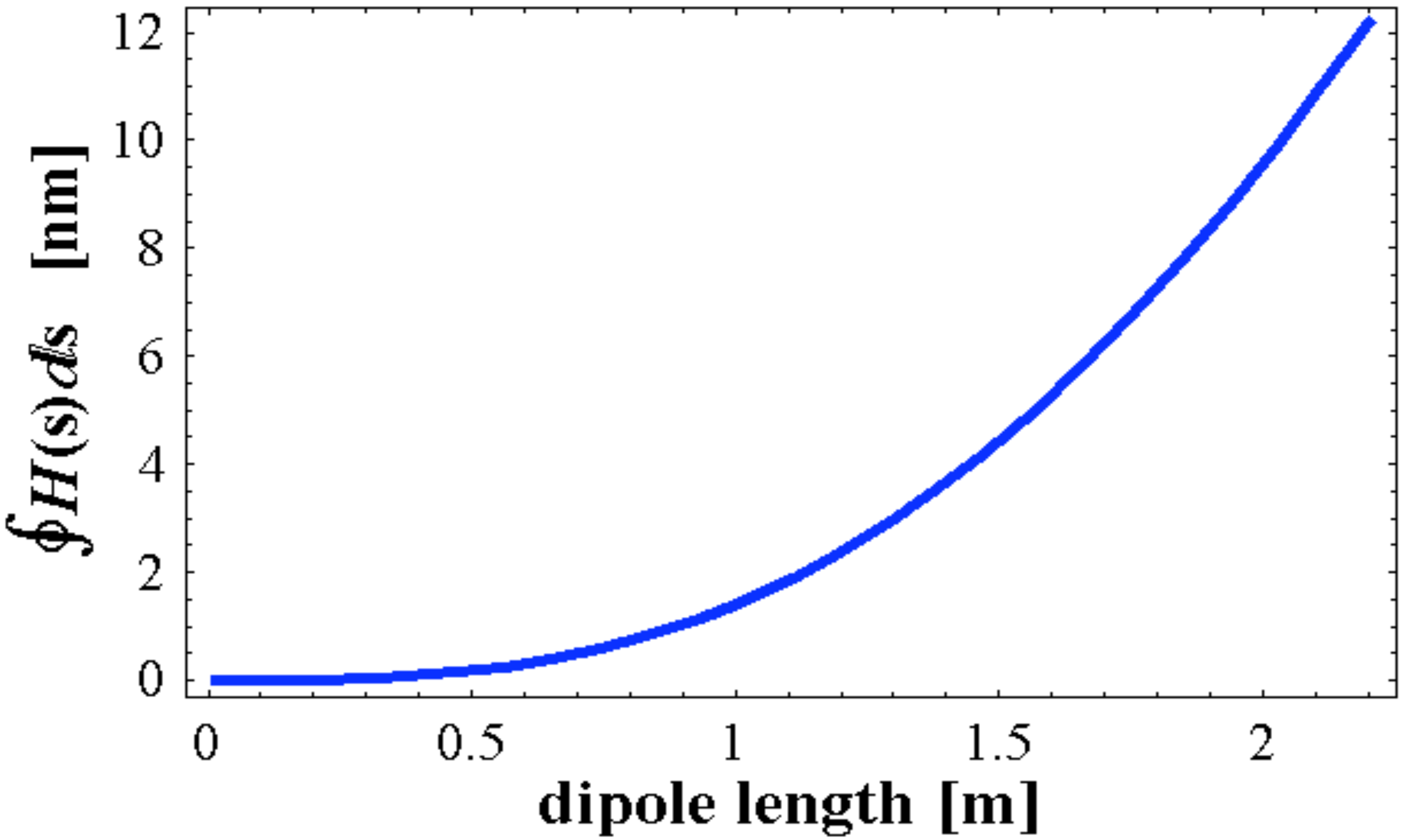
Longitudinally varying dipole fields

(Wrulich 1992, Guo and Raubenheimer 2002, Nagaoka 2004)

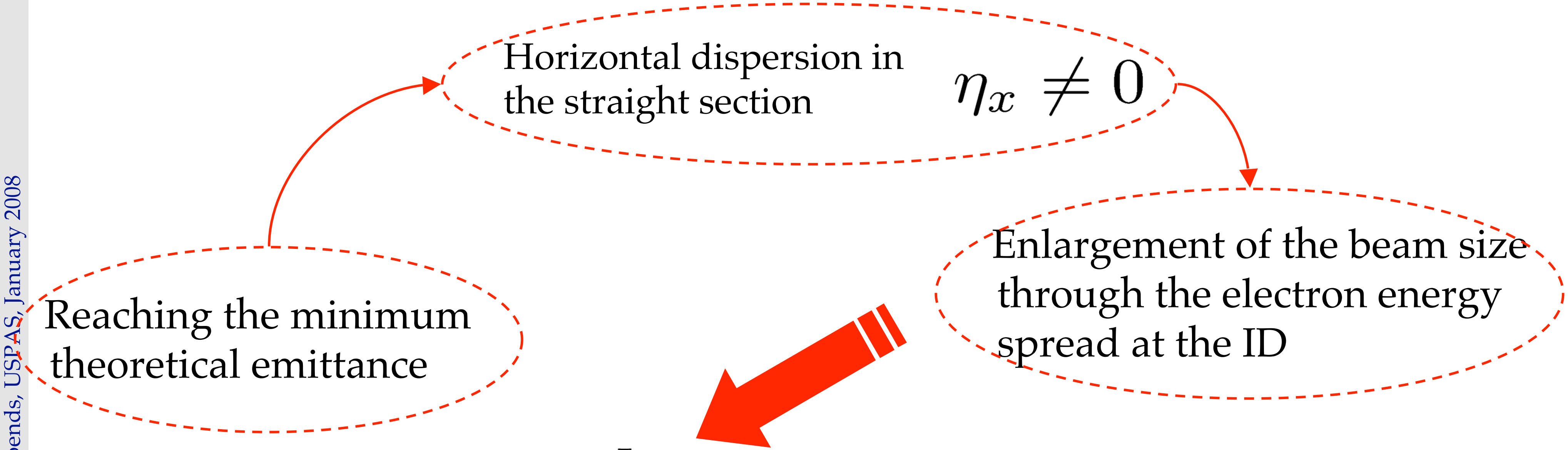
- For isomagnetic lattices, the minimum effective emittance depends on the integral

$$\frac{\oint \mathcal{H}_x(s)ds}{\rho_x} \propto \theta^3 = \frac{l_d^3}{\rho_x^3}$$

- Increase bending radius (i.e. decrease dipole field) where $\mathcal{H}_x(s)$ is high and vice-versa



The notion of effective emittance



The brilliance $\tilde{B} \propto \dfrac{I}{\epsilon_{x_{eff}}(s_{ID})\epsilon_{y_{eff}}(s_{ID})}$ is inversely proportional to

the **effective emittance** $\epsilon_{x;eff}(s)^2 \equiv \langle x(s)^2\rangle\langle x'(s)^2\rangle - \langle x(s)x'(s)\rangle^2$.

After replacing the expressions for position and angles and consider that the alpha function and dispersion derivative are zero on the ID

$$\boxed{\epsilon_{x_{eff}}(s_{ID}) = \sqrt{\epsilon_x^2 + \mathcal{H}_x(s_0)\epsilon_x\sigma_\delta^2}}$$

Effective emittance reminder

$$\epsilon_{x_{eff}}(s_{ID}) = \sqrt{\epsilon_x^2 + \mathcal{H}_x(s_0)\epsilon_x\sigma_\delta^2}$$

$$\mathcal{H}_x(s) = \beta_x(s)\eta_x'^2(s) + 2\alpha_x(s)\eta_x(s)\eta_x'(s) + \gamma_x(s)\eta_x^2(s)$$

"Phase space invariant"

$$\epsilon_x = \frac{C_q\gamma^2}{J_x}\frac{\oint \frac{\mathcal{H}_x(s)}{|\rho_x|^3}ds}{\oint \frac{1}{\rho_x^2}ds}$$

Equilibrium betatron emittance

Radiation integrals: $\mathcal{I}_{5_x}$, $\mathcal{I}_3$, $\mathcal{I}_2$, $\mathcal{I}_{4_x}$

$$\sigma_\delta^2 = \frac{C_q\gamma^2}{J_s}\frac{\oint \frac{1}{|\rho_x|^3}ds}{\oint \frac{1}{\rho_x^2}ds}$$

Equilibrium energy spread

$$J_x = 1 - \frac{\oint \frac{\eta_x(s)}{\rho_x^3}(1+2k\rho_x^2)ds}{\oint \frac{1}{\rho_x^2}ds} \quad , J_y = 1 \quad , J_s = 4 - J_x - J_y$$

Damping partition numbers

- Consider the transport matrix of a generalized dipole magnet with varying bending radius, in thin lens approximation and ignoring edge focusing

$$\mathcal{M}_{bend} = \begin{pmatrix} 1 & s & \widetilde{\theta(s)} \\ 0 & 1 & \theta(s) \\ 0 & 0 & 1 \end{pmatrix} \qquad \theta(s) = \int_0^s \frac{ds'}{\rho(s')} \ , \quad \widetilde{\theta(s)} = \int_0^s \theta(s')ds'$$

- At its entrance (from the ID side) the initial optics functions are

$$\beta_0 \ , \ \alpha_0 \ , \ \gamma_0 \ , \ \eta_0 \ , \ \eta_0'$$

and their evolution along the magnet is given by

$$\begin{aligned} \beta(s) &= \beta_0 - 2s\alpha_0 + s^2\gamma_0 \\ \alpha(s) &= \alpha_0 - s\gamma_0 \\ \gamma(s) &= \gamma_0 \\ \eta(s) &= \eta_0 + s\eta_0' + \widetilde{\theta(s)} \\ \eta(s) &= \eta_0' + \theta(s) \end{aligned}$$

- The transverse emittance is

$$\epsilon_x = \frac{C_q \gamma^2}{J_x \oint \frac{1}{\rho_x^2} ds} \left[\beta_0 (A3 + 2A2{\eta_0}' + A1{\eta_0}'^2) + \alpha_0 (2A5 + 2A4{\eta_0}' + \eta_0 (2A2 + 2A1{\eta_0}')) + \gamma 0 (A6 + 2A4\eta_0 + A1{\eta_0}^2) \right]$$

$\rightarrow C$

with
$$A1 = \oint \frac{1}{|\rho^3|} ds \;, A2 = \oint \frac{\theta(s)}{|\rho^3|} ds \;, A3 = -\oint \frac{\theta(s)^2}{|\rho^3|} ds$$

$$A4 = \oint \frac{\widetilde{\theta(s)} - s\theta(s)}{|\rho^3|} ds \;, A5 = \oint \frac{(\widetilde{\theta(s)} - s\theta(s))^2}{|\rho^3|} ds \;, A6 = -\oint \frac{\theta(s)(\widetilde{\theta(s)} - s\theta(s))}{|\rho^3|} ds$$

- By setting $J_s = 2J_x$ we get an expression of the effective emittance at the ID, depending on the initial optics functions

$$\epsilon_{x_{eff}}^2 = \frac{C}{2} \left[\gamma_0 (A6 + 2A4\eta_0 + A1\eta_0^2) + 2\alpha_0 (A5 + A2\eta_0 + A4\eta_0' + A1\eta_0\eta_0') + \beta_0 (A3 + 2A2\eta_0' + A1{\eta_0'}^2) \right]$$
$$\left[\gamma_0 (2A6 + 4A4\eta_0 + 3A1\eta_0^2) + 2\alpha_0 (2A5 + 2A2\eta_0 + 2A4\eta_0' + 3A1\eta_0\eta_0') + \beta_0 (2A3 + 4A2\eta_0' + 3A1{\eta_0'}^2 \right.$$

- The conditions for minimum effective emittance are

$$\frac{\partial \epsilon^2_{x_{eff}}}{\partial \eta_0} = 0 \ , \ \frac{\partial \epsilon^2_{x_{eff}}}{\partial \eta'_0} = 0 \ , \ \frac{\partial \epsilon^2_{x_{eff}}}{\partial \beta_0} = 0 \ , \ \frac{\partial \epsilon^2_{x_{eff}}}{\partial \alpha_0} = 0$$

- After some lengthy manipulations and exploiting certain symmetries of the equations, we obtain the following relations

$$\eta_0 = \frac{A4}{A2}\eta'_0 \ , \gamma_0 = \frac{A2(A3 + A2\eta'_0)\beta_0}{2A2A6 + A4^2\eta'_0} \ , \alpha_0 = -\frac{A2(A5 + A4\ \eta'_0)\beta_0}{2A2A6 + A4^2\eta'_0}$$

- Finally, one has to solve the following equation for the dispersion

$$3{\eta'_0}^3 + 10T1{\eta'_0}^2 + T1^2(6 - 5T2)\eta'_0 - 4T1^3T2 = 0$$

$$T1 = \frac{A2}{A1} \qquad T2 = \frac{A1(A5^2 - A3A6)}{A3A4^2 + A2(-2A4A5 + A2A6)}$$

- Keeping the real solution of the 3rd order polynomial equation, and replacing in the previous conditions, we obtain the optics functions for minimum effective emittance

$$\beta_0 = \frac{9A1A6T + A4^2(46+(-10+T)T+45T2)}{3\sqrt{A1A3(A4^2+A2(-2A4A5+A2A6))T(46+T(-10+T-9T2)+45T2}} ,$$

$$\alpha_0 = -\frac{A1(9A5T + A4T1(46+(-10+T)T+45T2)}{3\sqrt{A1(A4^2+A2(-2A4A5+A2A6))T(46+T(-10+T-9T2)+45T2}} ,$$

$$\eta_0 = \frac{A4(-10+T+\frac{46+45T2}{T})}{9A1} ,$$

$$\eta'_0 = \frac{T1(-10+T+\frac{46+45T2}{T})}{9} ,$$

with $T = \left(-190-189T2+9\sqrt{-3(1+T2)(2+3T2)(126+125T2)}\right)^{\frac{1}{3}}$.

- By replacing, we get an analytic expression for the minimum effective emittance for any dipole field profile

$$\epsilon_{x_{eff}} = \frac{C}{9}\sqrt{\frac{S2(T^4-2T^3-6T^2(3T2-2)-2T(45T2+46)+(45T2+46)^2)(T^4+7T^3-6T^2(12T2+13)+7T(45T2+46)+(45T2+46)^2)}{6A1T^3(46+T(-10+T-9T2)+45T2)}}$$

Special cases

- In the case of **constant field** we obtain the relation of Tanaka and Ando (1996)

$$\epsilon_{x;eff_{min}} = 0.03339 C_q \frac{\gamma^2\theta^3}{J_x}$$

which is a factor of **1.55** higher than the minimum betatron emittance

$$\epsilon_{x_{min}} = \frac{1}{12\sqrt{15}} C_q \frac{\gamma^2\theta^3}{J_x}$$

- For an ESRF Double Bend lattice (64 dipoles, 6GeV), the minimum effective emittance is **1.69nm**
- Setting $A = (2A_2A_4 - A_1A_5)A_5 - A_2^2A_6 + A_3(A_1A_6 - A_4^2)$,

the **minimum betatron emittance** is obtained for the optics function conditions

$$\epsilon_{x;min} = \frac{2C\sqrt{A_1A}}{A_1}$$

$$\eta_0 = -\frac{A_4}{A_1} \;, \beta_0 = \frac{-A_4^2 + A_1A_6}{\sqrt{A_1A}} \;, \eta_0' = -\frac{A_2}{A_1} \;, \alpha_0 = \frac{A_2A_4 - A_1A_5}{\sqrt{A_1A}} \;.$$

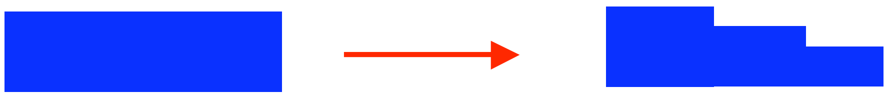
- Imposing **achromatic conditions** $\eta_0 = \eta_0' = 0$

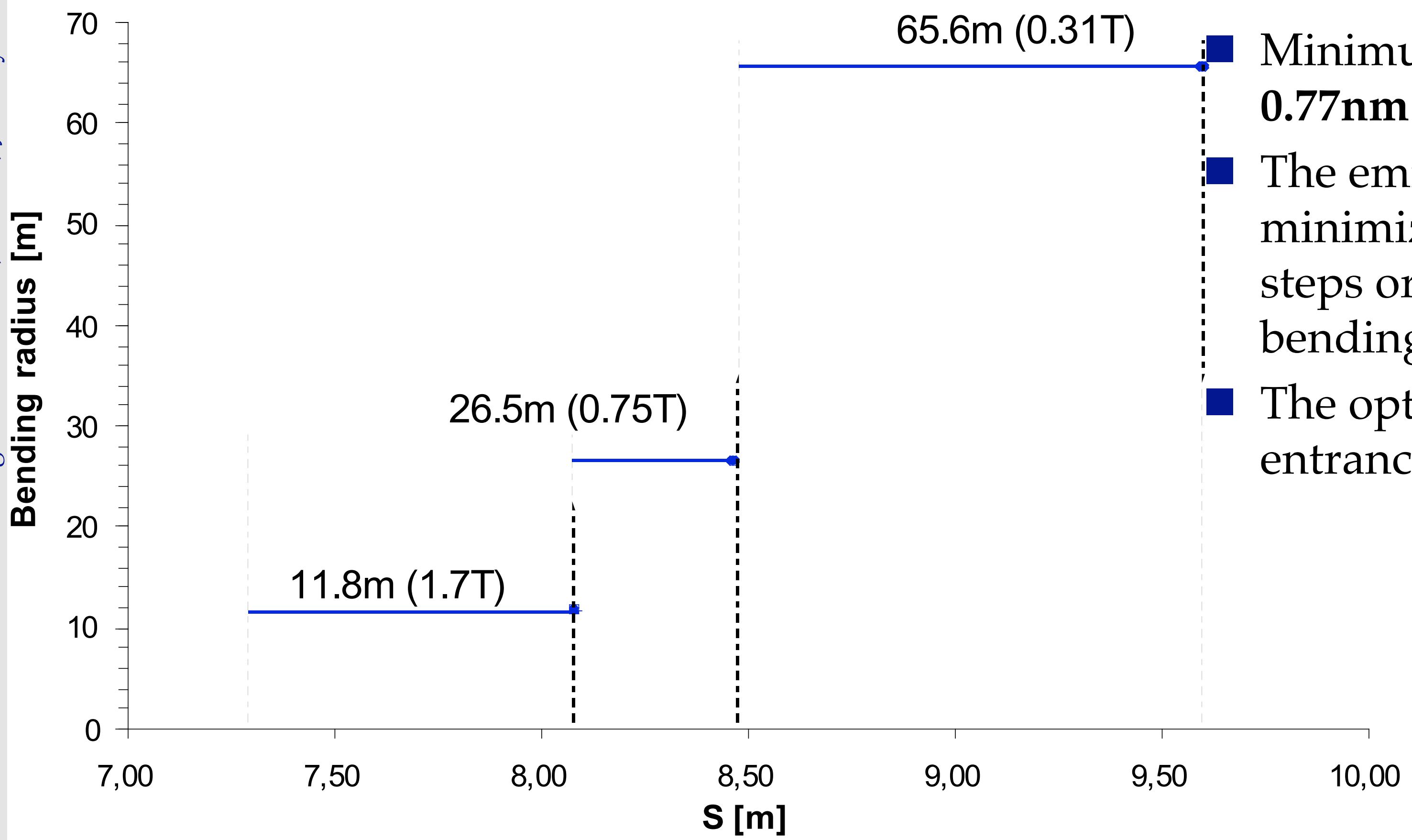
the minimum betatron (=effective) emittance is obtained for the optics function conditions

$$\epsilon_{x;min} = 2C\sqrt{A_3A_6 - A_5^2}$$

$$\beta_0 = \frac{A_6}{\sqrt{A_3A_6 - A_5^2}} \;, \alpha_0 = \frac{A_5}{\sqrt{A_3A_6 - A_5^2}} \;.$$

Variable bend with three steps





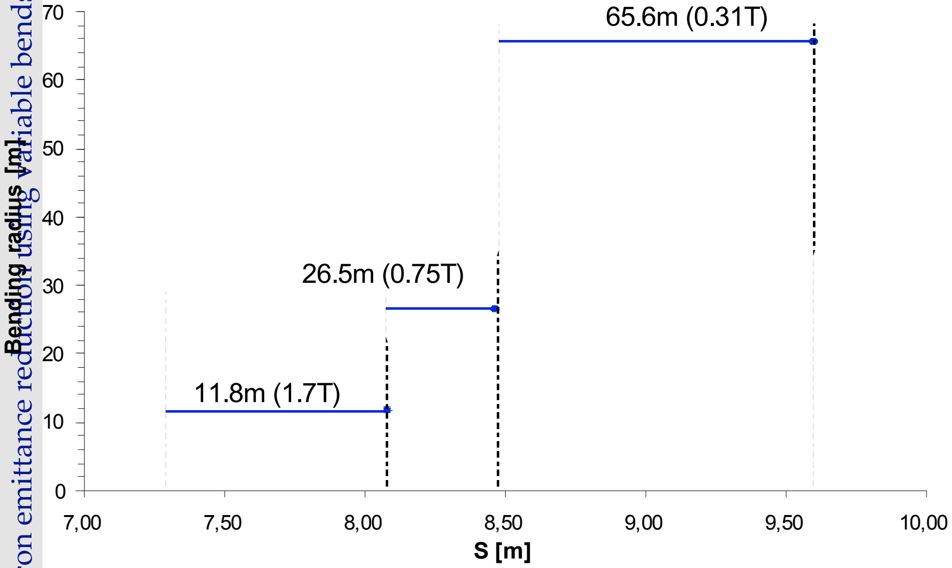
- Simple field configuration (see ESRF dipole with soft edges)
- Maximum of **1.7 T** to avoid saturation
- Minimum effective emittance of **0.77nm** obtained
- The emittance can be further minimized by adding more steps or raising the maximum bending field
- The optics function, at the entrance, for this configuration

$$\beta_0 = 1.23 \text{ m},$$
$$\alpha_0 = 2.76,$$
$$\eta_0 = 0.008 \text{ m},$$
$$\eta'_0 = -0.030$$

Equilibrium energy spread in a DVB

$$\sigma_\delta^2 = \frac{C_q \gamma^2}{J_s} \frac{\oint \frac{1}{|\rho_x|^3} ds}{\oint \frac{1}{\rho_x^2} ds}$$

(circled term $\frac{C_q \gamma^2}{J_s}$ → c_1)



For a uniform field dipole

$$\sigma_\delta = \sqrt{\frac{c_1}{\rho_x}} = \sqrt{c_1 \frac{\theta}{l}}$$

For the Variable 3-step bend

$$\rho_1 \approx 2\rho \ , \ \rho_2 \approx \rho \ , \ \rho_3 \approx \rho/2$$

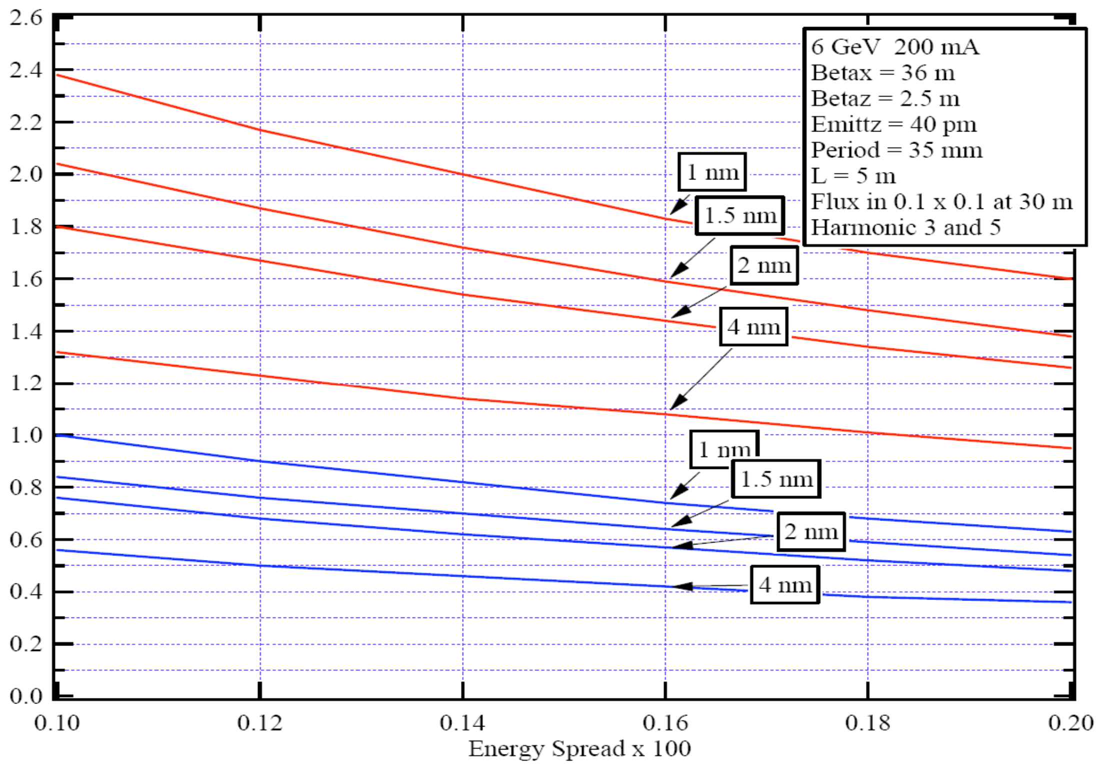
and $l_1 \approx l/3 \ , \ l_2 \approx l/12 \ , \ l_3 \approx l/2$

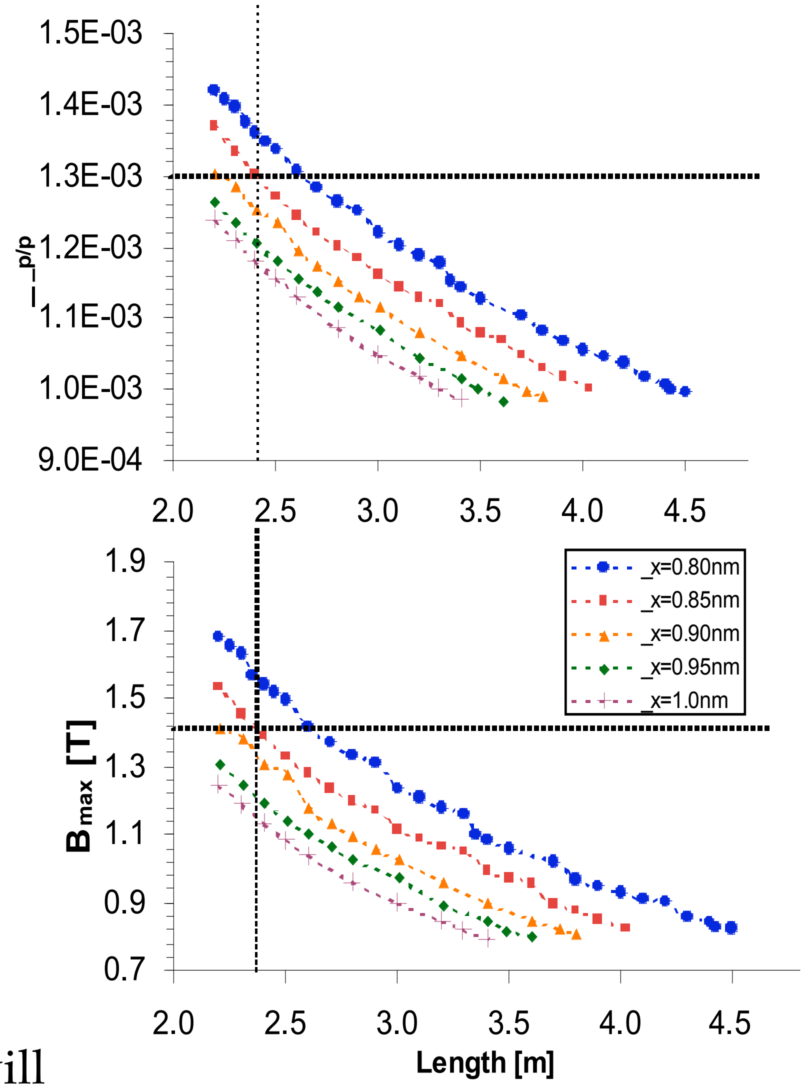
The energy spread is

$$\sigma_\delta \approx \frac{3}{2}\sqrt{\frac{11}{13}} \sigma_{\delta_{ESRF}} = 1.4610^{-3}$$

Taking the uniform field approximation this implies that for having the same energy dispersion $l_{tot} \approx \frac{99\, l}{52} = 1.9 l \approx 4.4 m$ and the max. field should drop accordingly

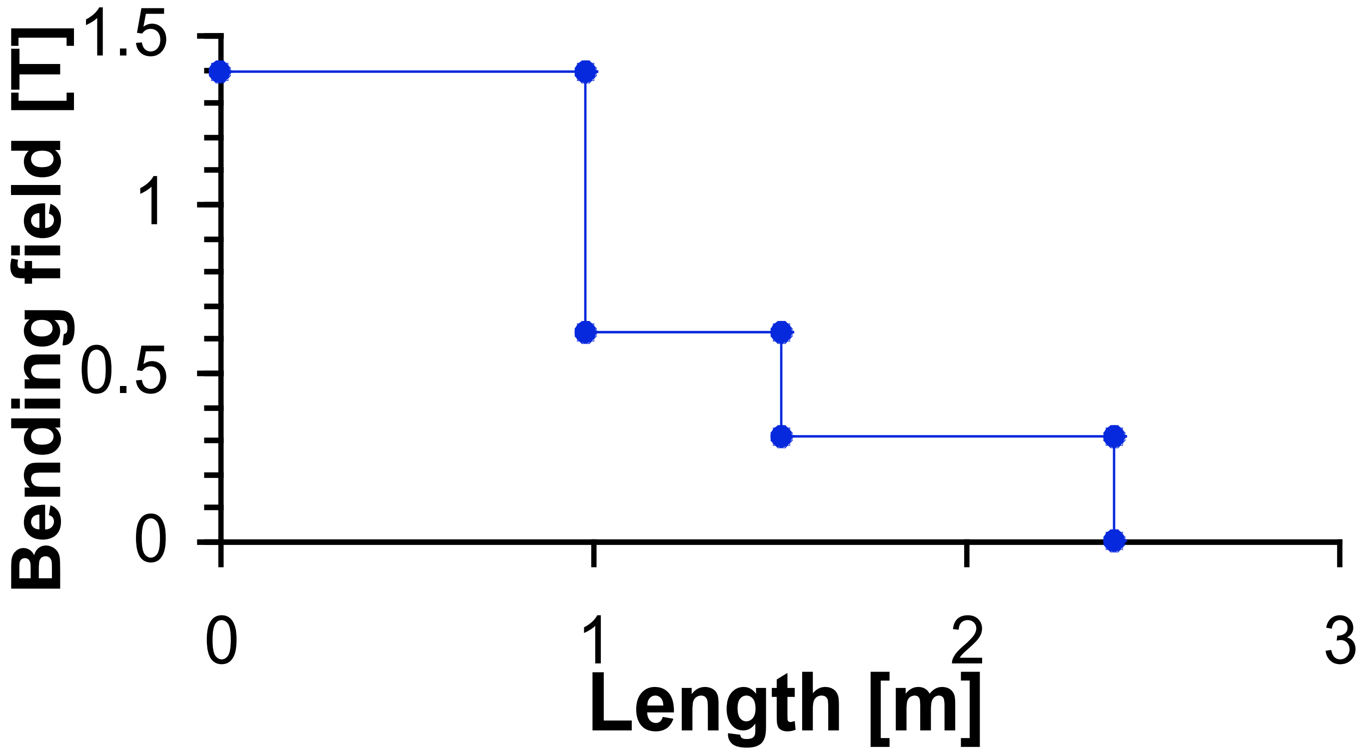
Constraining the dipole field





- We choose **1.3e-3** as the target energy spread (**13%** reduction in the flux for harmonic **3 at 1nm**)
- A fixed energy spread and a dipole length of **2.4m** will impose the maximum field (**1.4T**) and the minimum emittance

Choosing the variable dipole



- Minimum emittance achieved of **0.85nm**
- Maximum field of **1.4T**
- Initial optics functions are $\beta_0 = 1.49$ m, $\alpha_0 = 2.6$, $\eta_0 = 0.011$ m, $\eta'_0 = -0.031$

compared to for the extreme DVB (0.77nm) $\beta_0 = 1.23$ m, $\alpha_0 = 2.76$, $\eta_0 = 0.008$ m, $\eta'_0 = -0.030$

and for the actual SR $\beta_0 = 1.79$ m, $\alpha_0 = 1.39$, $\eta_0 = 0.073$ m, $\eta'_0 = -0.080$

- Note that beta at the dipole exit is 19m

Phase advance for minimum effective emittance cell

- **General rule**: Provided that dispersion is not zero, there is a **unique** phase advance for a straight section with mirror symmetry in the center

- Given the initial (final) optics functions $\beta_0,\ \alpha_0,\ \eta_0,\ \eta'_0$ the phase advance for such a line is

$$\tan(\mu) = \frac{2\eta_0(\beta_0\eta'_0 + \alpha_0\eta_0)}{(\beta_0\eta'_0 + (\alpha_0 - 1)\eta_0)(\beta_0\eta'_0 + (\alpha_0 + 1)\eta_0)}$$

- Applying the result to an arbitrary double bend cell, we obtain

$$\mu_{cell} = \mu_{cell}(\beta_0, \alpha_0, \eta_0, \eta'_0, l_d, \theta, \tilde{\theta})$$

a function depending **only** on the initial optics functions and the dipole !!!

- The horizontal phase advance for reaching the absolute minimum effective emittance at the ESRF storage ring is **293**^{o} (**205**^{o} actually)
- The horizontal phase advance for reaching the effective emittance minimum for the three step double variable bend lattice is **355**^{o}

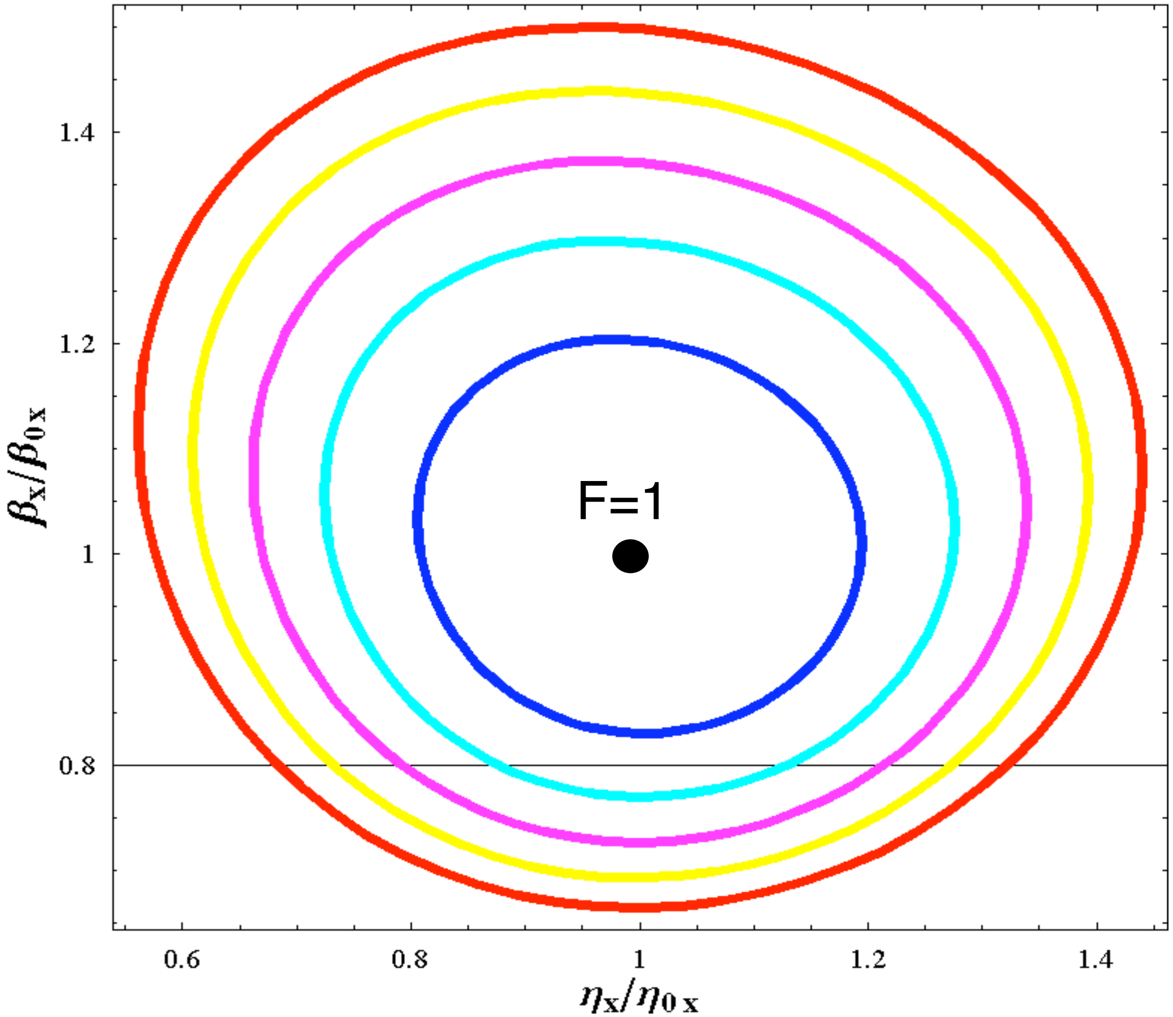
Emittance ratio for detuned optics functions

- By detuning the initial beta and dispersion we obtain curves of equal effective emittance ratio

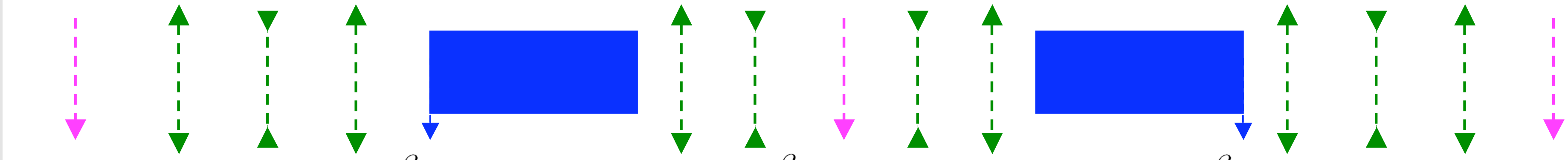
$$F = \frac{\epsilon_{x_{eff}}}{\epsilon_{x_{eff_{min}}}}$$

- Poss 4-parametric plot for all optics functions
- Note that by detuning the optics functions, the phase advance also changes (**lower** for **higher** F values)

(Emma and Raubenheimer 2001, Streun 2001, Korostelev and Zimmermann 2003)



Constraints for general double bend cells



β_{ID1}, $\alpha_{ID1} = 0$, $\eta_{ID1} = \sqrt{\beta_{ID1}\,\mathcal{H}}$, $\eta'_{ID1} = 0$

β_0, α_0, η_0, η'_0

β_{SP}, $\alpha_{SP} = 0$, $\eta_{SP} = \sqrt{\beta_{SP}\,\mathcal{H}}$, $\eta'_{SP} = 0$

β_0, α_0, η_0, η'_0

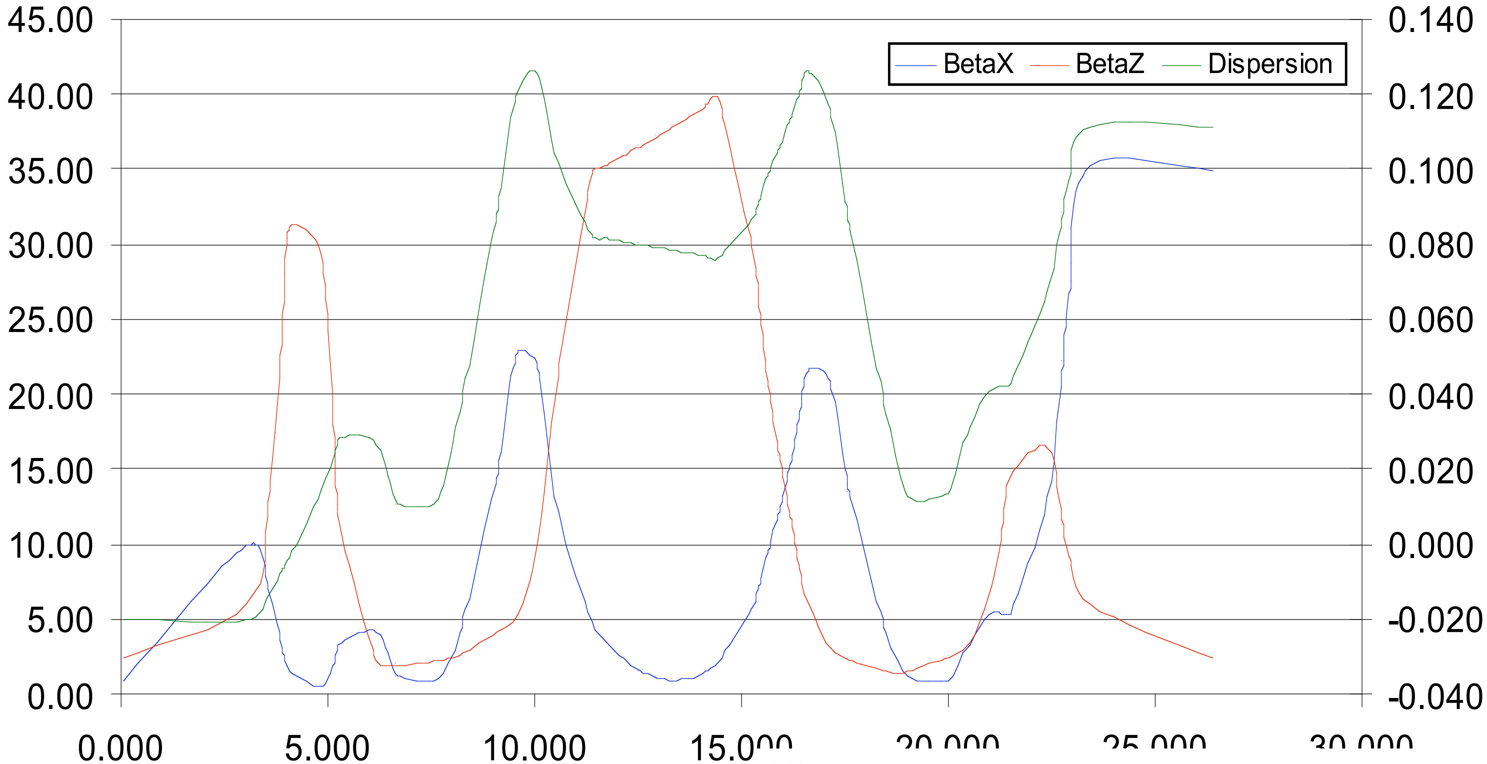
β_{ID2}, $\alpha_{ID2} = 0$, $\eta_{ID2} = \sqrt{\beta_{ID2}\,\mathcal{H}}$, $\eta'_{ID2} = 0$

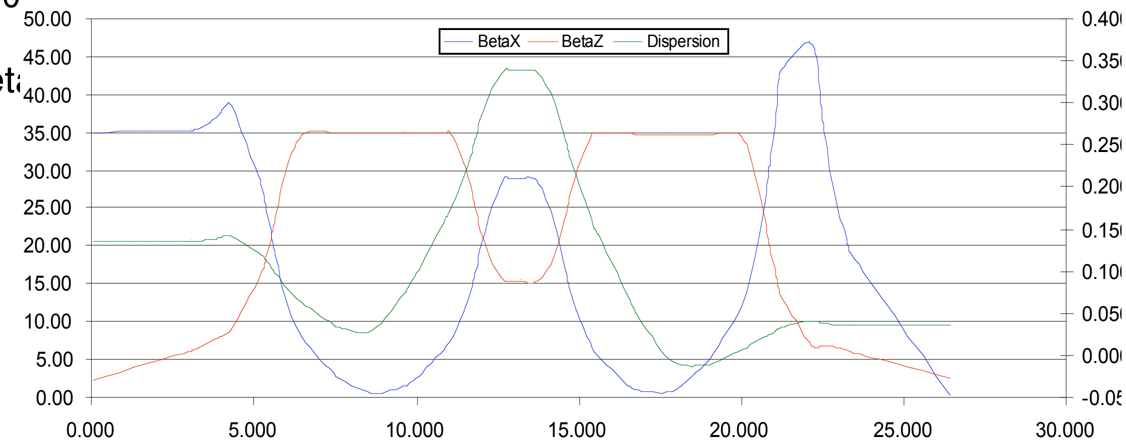
- Consider a general double bend with the ideal effective emittance (drifts are parameters)
- In the **straight section** between the ID and the dipole entrance, there are **three constraints**, thus at least **three quadrupoles** are needed
- In the **"achromat"**, there are **two constraints**, thus at least **two quadrupoles** are needed (one and a half for a symmetric cell)
- Note that there is **no control** in the vertical plane
- The vertical phase advance is also **fixed**!!!!
- Expressions for the quadrupole gradients can be obtained, parameterized with the drift lengths, the initial optics functions and the beta on the IDs
- All the optics functions are thus uniquely determined for both planes and can be minimized (the gradients as well) by varying the drifts
- The **chromaticities** are also **uniquely defined**

Constraints for a Double Variable Bend structure @ the ESRF

- Constraints for the dipole
  - Energy of **6GeV**, **64** dipoles, i.e. total bending radius of **π/32**
  - Dipole length of **2.3m**
  - Maximum dipole field of **1.4T** (imposed by momentum spread of 1.3e-3)
- Constraints for the drifts
  - Cell length of **26.4m**
  - ID drift of **3m** → vertical beta of **2.5m** at the ID
  - Drift next to dipoles ρ **0.5m** (space for the absorber)
  - Drifts between quadrupoles ρ **0.5m** (space for sextupoles, correctors, BPM, etc.)
- Constraints for the quadrupoles
  - Maximum gradient of **45T/m** (reducing the bore diameter by a factor of 2)
- Constraints for the sextupoles
  - Maximum integrated sextupole strength of **35m^{-2}** (Master thesis of T. Perron 2002)
- Constraints for the optics functions

Extreme DVB with low energy spread





- Effective Emittance of **0.96nm** (0.95nm in the high beta and 0.97nm in the low beta)
- Max. quad strength of **45T/m** (15 T/m for the SR
- Max. betas of **35 and 40m** (46 and 35m for the SR)
- Maximum dispersion of **0.13m** (0.34m for the SR)
- Chromaticities of **(-169, -160)** (-132, -50 for the SR)
- Phase adv. of **(357°,166°)** (205°,81° for the SR)

High phase advance implications

Strong quadrupoles

Big phase advance
μ_x

High Chromaticity
$\xi_{x,y}(s) \propto K(s)\beta_{x,y}(s)$

Small dispersion
$\eta(s) \approx \sqrt{\frac{a_C R}{\nu_x}}\sqrt{\beta_x(s)}$

Strong sextupoles
$\xi_{x,y}(s) \propto S(s)\eta(s)\beta_{x,y}(s)$

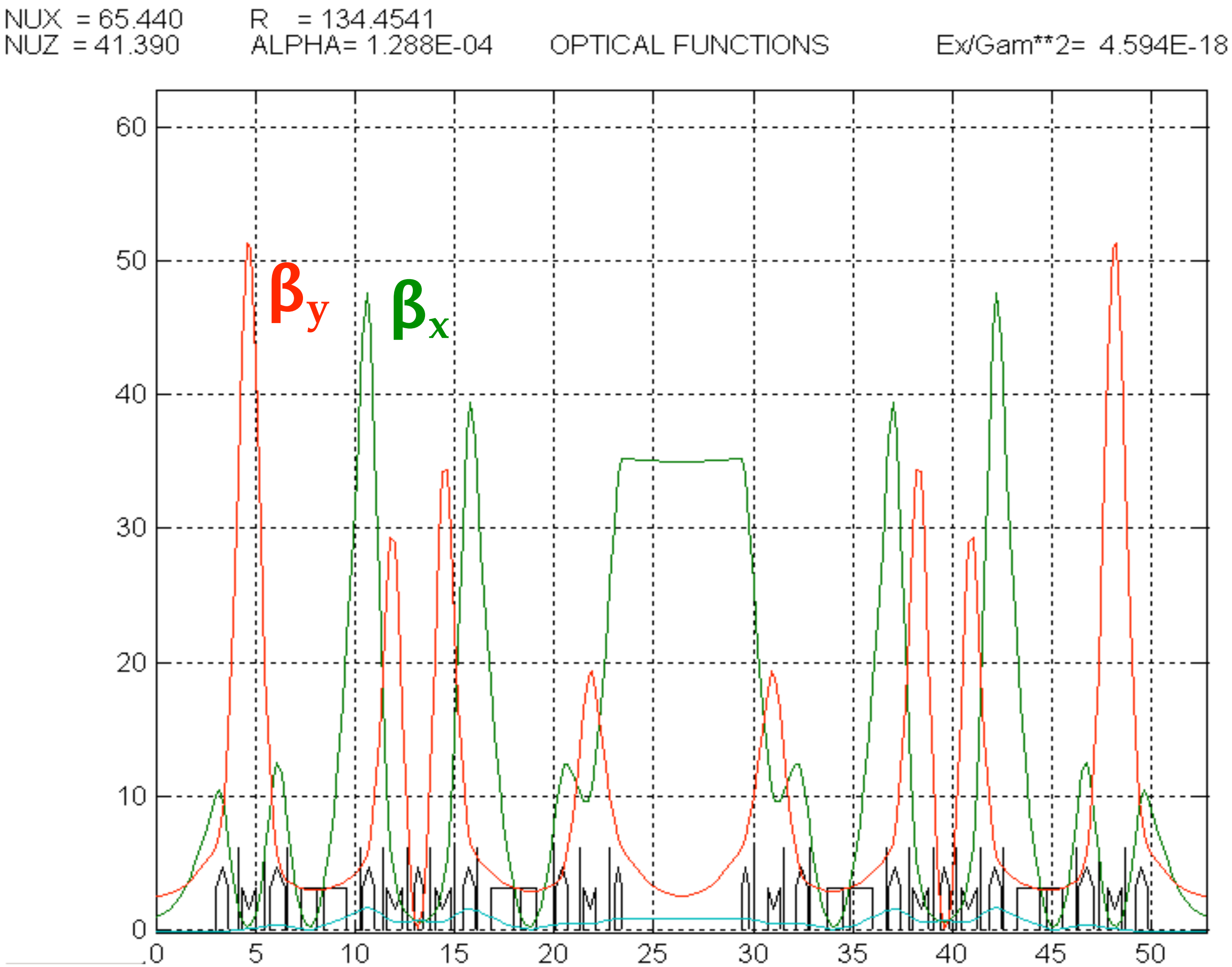
Small dynamic aperture

Limited injection aperture

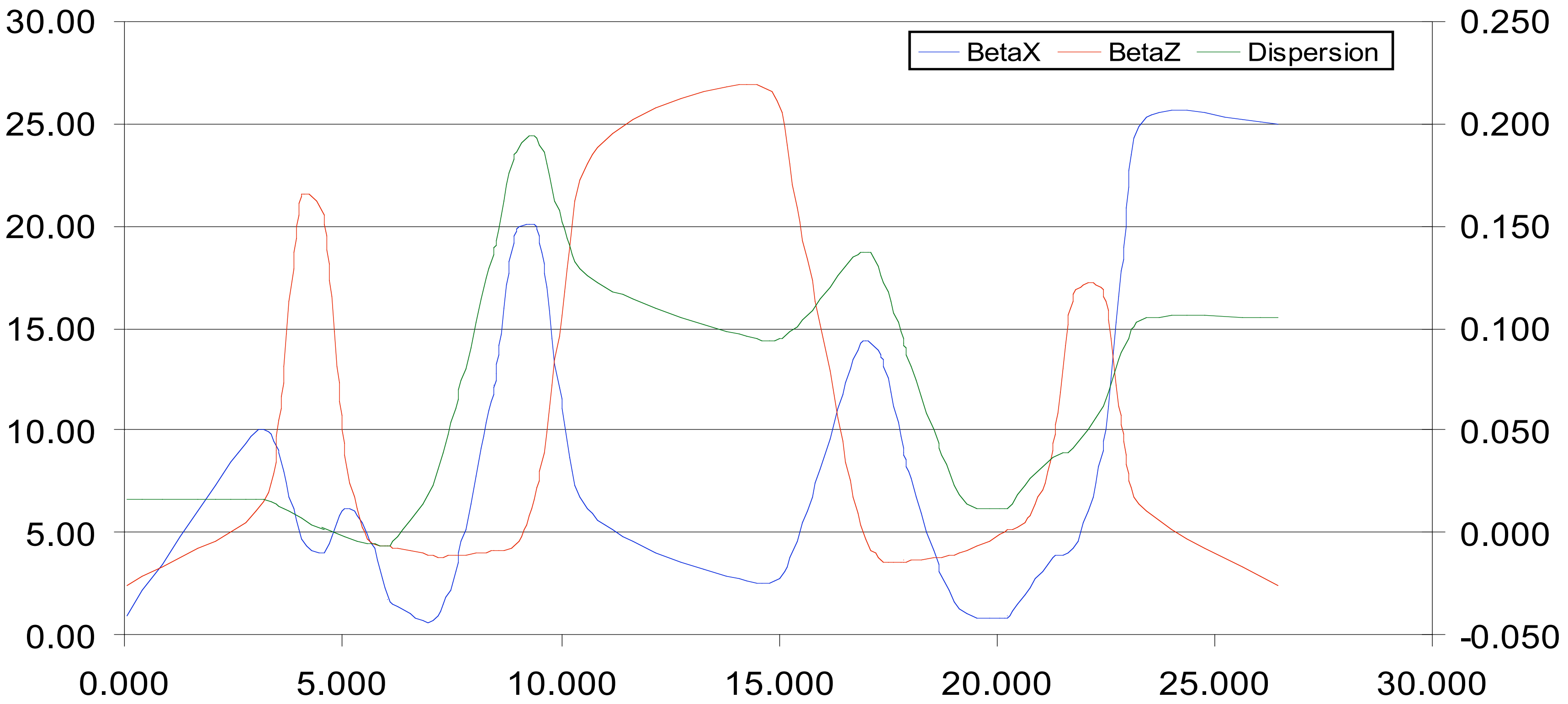
Limited lifetime

Some comments…

- The maximum quad length is of **0.9m**
- The distance between the dipoles and quads is **0.5m** (min. distance allowed between dipoles and quads)
- The distance between the quads in the middle of the "achromat" is bigger than **3m**
- In that area, the hor. beta is small (only efficient for vertical chromaticity correction)
- This space can be occupied by another dipole or ID element (convergence between TBA and DVB solution)
- Preliminary non-linear optimisation showed poor DA



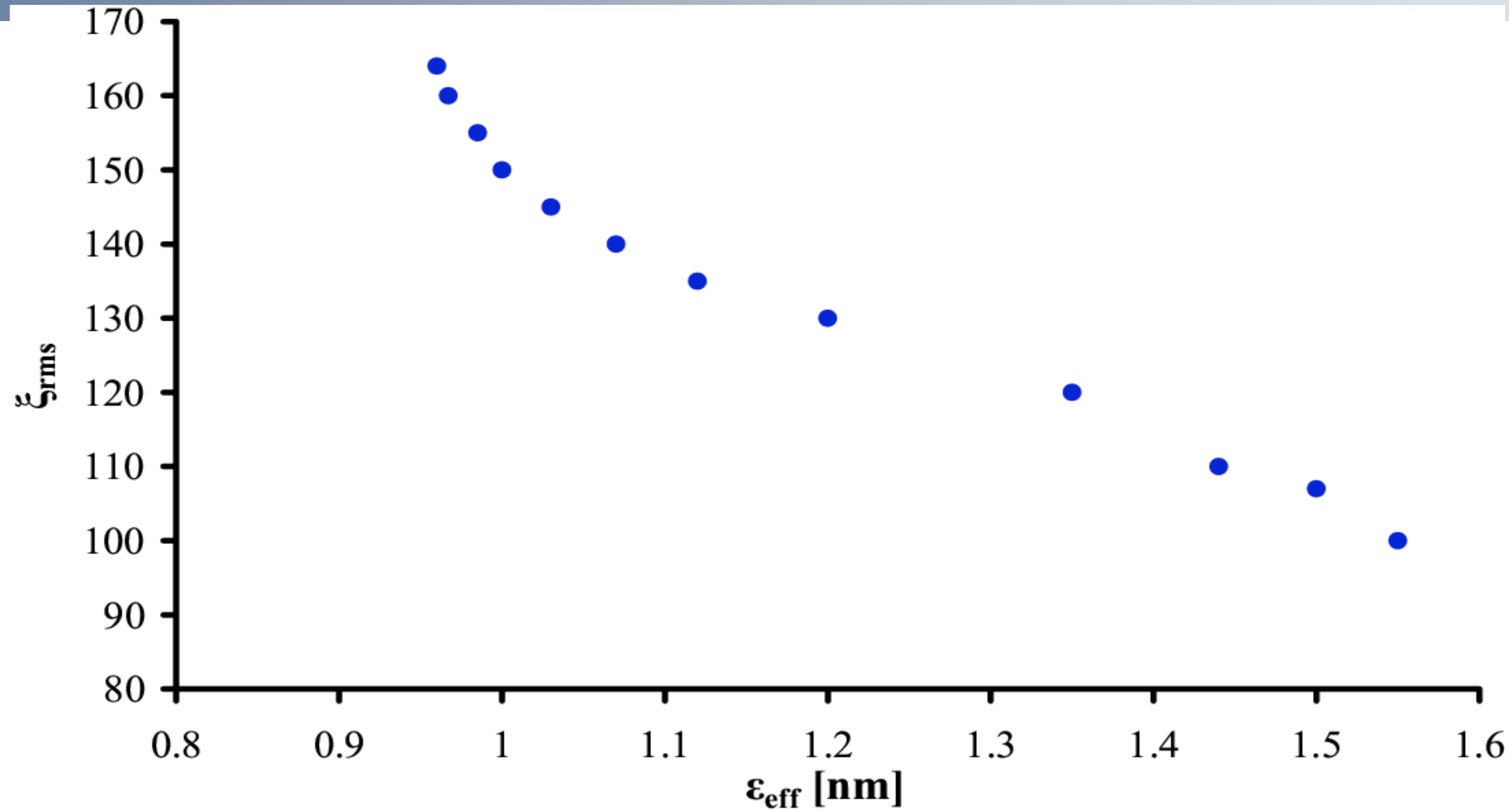
Relaxed DVB with low energy spread



- Effective Emittance of **1.55nm** (1.5nm in the high beta and 1.61nm in the low beta) (compared to 0.96)
- Max. quad strength of **46T/m** (compared to 45 T/m)
- Max. betas of **35 and 40m** (compared to 35 and 40 m)
- Maximum dispersion of **0.19m** (compared to 0.13m)
- Chromaticities of **(-110, -89)** (compared to -169, -160)

- Phase adv. of **(275°,129°)** (compared to 357°,166°)
- The maximum quad length is of **0.8m**
- The distance between the dipoles and quads is **0.5m**
- The distance between the quads in the middle of the "achromat" is **3.8m**, with the same low hor.beta
- Preliminary runs show a horizontal DA of around **30mm** (target value is 20mm imposed by injection aperture)

Scaling of the chromaticity with emittance



- Emittance scales almost linearly with chromaticity.
- Question to be answered: lowest emittance that can be achieved which leading to a reasonable DA.
- Preliminary scaling suggests that this emittance may be found around **1.3nm**
- Top-up could allow a small of momentum DA (lifetime), at least **10mm** are mandatory for ensuring efficient injection.

Upgrade stages

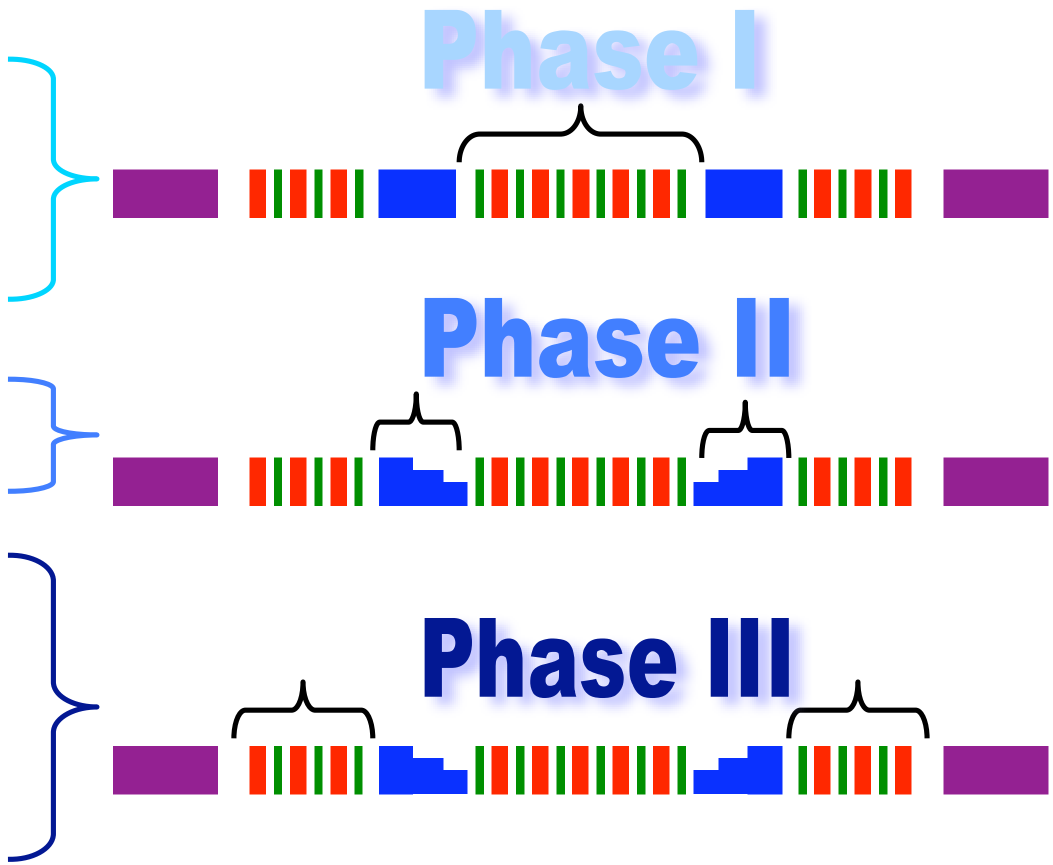
Ultimate lattice drawbacks

- Long interruption time for installation of all components
- Long commissioning to reach ultimate performance (2-3 years)

- Changing half of each cell (achromat)
- Increase the phase advance to reach **2nm**
- Increase the current to **300mA** (feed-back)
- **3-fold increase** of brilliance

- All dipoles replaced by variable bends
- Small gain in emittance

- All straight section magnets are replaced
- **Sub-nanometer** emittance
- An RF upgrade to reach more than **500mA**
- Brilliance increased by a **factor of 10**



Design challenges

- Adequate **dynamic aperture** for high phase advance cells
- Variable **bending magnets field quality**
- Building **high gradient quadrupoles** with incorporated sextupole components
- Design of new **absorbers** to sustain high beam power due to current upgrade
- High-gradient magnets need low gaps and small vacuum chambers, i.e. **impedance increase** (NEG coating)
- Design of **septum** with smaller sheet thickness
- Optimising injection process (booster, transfer lines) to allow continuous **top-up operation**