Linear imperfections and correction

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- Steering error and closed orbit distortion
- Gradient error and beta beating correction
- Linear coupling and correction
- Chromaticity

Beam stability in storage rings

- Beam orbit stability very critical especially for the stability of the synchrotron light spot in the beam lines
- Consequences of orbit distortion
  - Miss-steering of photon beams, modification of the dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling of beam motions, modulation of lattice functions, poor injection efficiency
- Long term Causes (Years - months)
  - Ground settling, season changes, diffusion, medium - Days/Hours, sun and moon, day-night variations (thermal), rivers, rain, water table, wind, synchrotron radiation, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns
- Short (Minutes/Seconds)
  - Ground vibrations, power supplies, injectors, insertion devices, air conditioning, refrigerators/compressors,water cooling, beam instabilities in general

Closed orbit distortion

- Causes
  - Dipole field errors
  - Dipole misalignments
  - Quadrupole misalignments
    - Consider the displacement of a particle **δx** from the ideal orbit . The vertical field is

$$B_y = G\bar{x} = G(x+\delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

- Remark: Dispersion creates a closed orbit distortion for off-momentum particles $\delta x = D(s)\frac{\delta p}{p}$
- Effect of orbit errors in any multi-pole magnet

$$B_y = b_n \bar{x}^n = b_n (x+\delta x)^n = b_n(\underbrace{x^n}_{\textbf{2(n+1)-pole}} + \underbrace{n\delta x x^{n-1}}_{\textbf{2n-pole}} + \underbrace{\frac{n(n-1)}{2}(\delta x)^2 x^{n-2}}_{\textbf{2(n-1)-pole}} + \cdots + \underbrace{(\delta x)^n}_{\textbf{dipole}})$$

- **Feed-down**

Effect of single dipole kick

- Introduce **Floquet variables**

$$\mathcal{U} = \frac{u}{\sqrt{\beta}} \ , \ \ \mathcal{U}' = \frac{d\mathcal{U}}{d\phi} = \frac{\alpha}{\sqrt{\beta}}u + \sqrt{\beta}u' \ , \ \ \phi = \frac{\psi}{\nu} = \frac{1}{\nu}\int \frac{ds}{\beta(s)}$$

- The Hill's equations are written $\frac{d^2\mathcal{U}}{d\phi^2} + \nu^2\mathcal{U} = 0$
- The solutions are the ones of an harmonic oscillator $\mathcal{U} = \mathcal{U}_0 \cos(\nu\phi)$
- Consider a single dipole kick $\delta u'(\pi) = \frac{\delta(Bl)}{B\rho}$ at **$\varphi=\pi$**
- Then

$$\mathcal{U}'(\pi) = -\mathcal{U}_0\nu\sin(\pi\nu) = \frac{d\mathcal{U}}{d\phi}\Big|_{\phi=\pi} = \frac{d\mathcal{U}}{ds}\frac{ds}{d\phi}\Big|_{s=k} = \frac{d\mathcal{U}}{ds}\Big|_{s=k}\beta(k)\nu = \sqrt{\beta(k)}\frac{du}{ds}\Big|_{s=k}$$

and $\mathcal{U}_0 = \frac{\sqrt{\beta(k)}}{2|\sin(\nu\pi)|}\delta u'(\pi)$ with $\frac{\delta u'(\pi)}{2} = \frac{du}{ds}\Big|_{s=k} = \frac{\delta(Bl)}{2B\rho}$

and in the old coordinates

$$u(s) = \sqrt{\beta(s)}\mathcal{U}_0\cos(\nu\phi(s)) = \underbrace{\frac{\sqrt{\beta(s)\beta(k)}}{2\sin(\pi\nu)}}\frac{\delta(Bl)}{B\rho}\cos(\nu\phi(s))$$

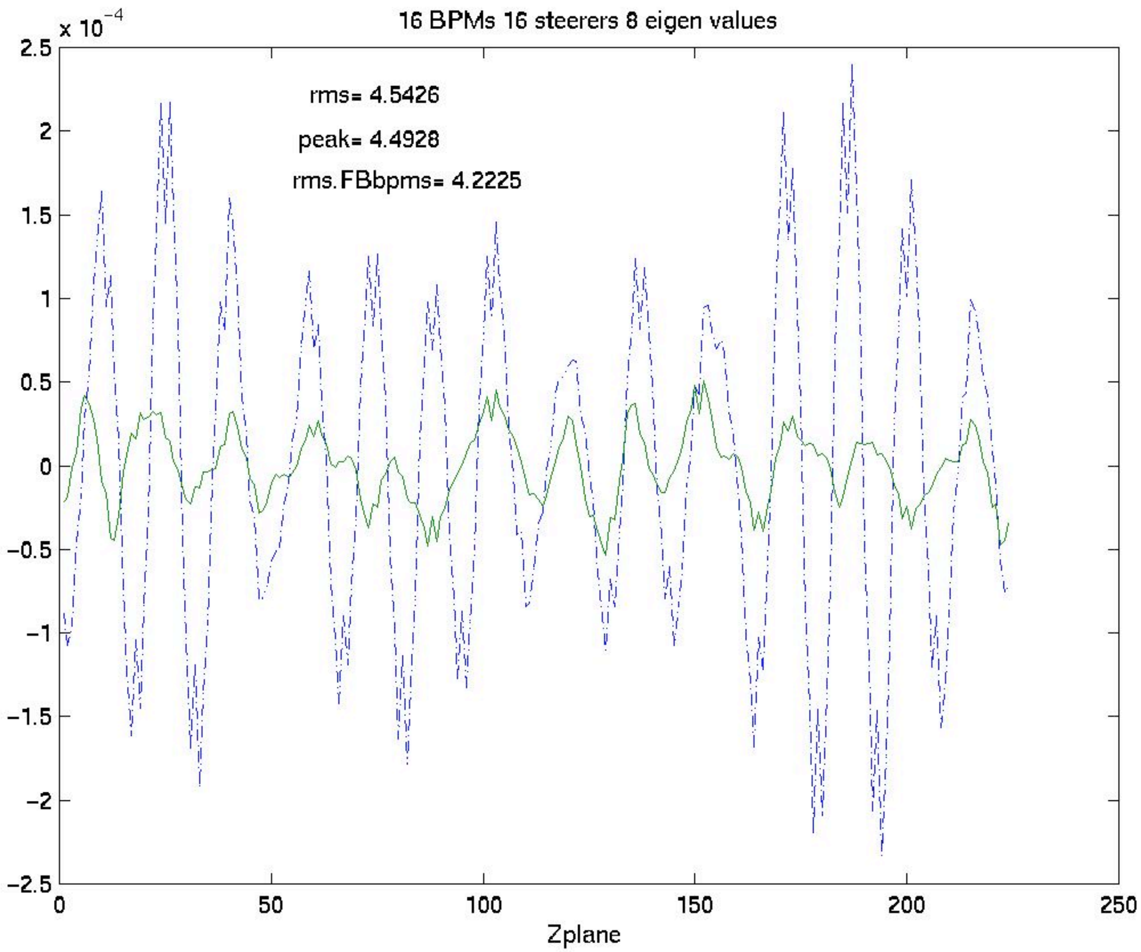
Maximum distortion amplitude

Example: Orbit distortion in ESRF storage ring

- In the ESRF storage ring, the beta function is **1.5m** in the dipoles and **30m** in the quadrupoles.
- Consider dipole error of **δy′=1mrad**
- The horizontal tune is **36.44**
- Maximum orbit distortion in dipoles

$$y_0 = \frac{\sqrt{1.5 \cdot 1.5}}{2\sin(36.44\pi)} \cdot 10^{-3} \approx 1\text{mm}$$

- For quadrupole displacement with **1mm**, the distortion is
- Magnet alignment is critical $y_0 \approx 8\text{mm}$!!!



Closed orbit distortion

Horizontal-vertical orbit distortion (Courant and Snyder 1957)

$$\delta_{x,y}(s) = -\frac{\sqrt{\beta_{x,y}}}{2\sin(\pi Q_{x,y})}\int_s^{s+C}\frac{\Delta B(\tau)}{B\rho}\sqrt{\beta_{x,y}}\cos(|\pi Q_{x,y}+\psi_{x,y}(s)-\psi_{x,y}(\tau)|)d\tau$$

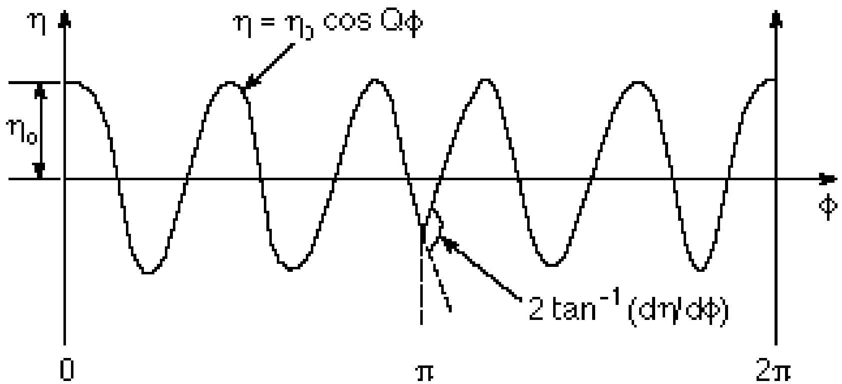
with $\Delta B(\tau)$ the equivalent magnetic field error at $s=\tau$.

Approximate errors as delta functions in n locations:

$$\delta_{x,y;i} = -\frac{\sqrt{\beta_{x,y;i}}}{2\sin(\pi Q_{x,y})}\sum_{j=i+1}^{i+n}\phi_{x,y;j}\sqrt{\beta_{x,y;j}}\cos(|\pi Q_{x,y}+\psi_{x,y;i}-\psi_{x,y;j}|)$$

with $\phi_{x,y;j}$ kick produced by jth element:

- $\phi_j = \frac{\Delta B_j L_j}{B\rho} \rightarrow$ dipole field error
- $\phi_j = \frac{B_j L_j \sin\theta_j}{B\rho} \rightarrow$ dipole roll
- $\phi_j = \frac{G_j L_j \Delta x, y_j}{B\rho} \rightarrow$ quadrupole displacement



Many orbit errors' effect

- Consider random distribution of errors in N magnets
- The expectation value is given by

$$< u(s) >= \frac{\sqrt{\beta(s)}}{2\sqrt{(2)}\sin(\pi\nu)} \sum_i \beta_i \delta u_i' = \frac{\sqrt{\beta(s)\langle\beta\rangle}\sqrt{N}}{2\sqrt{(2)}\sin(\pi\nu)} \frac{(\delta Bl)_{\rm rms}}{B\rho}$$

- Example:
  - In the ESRF storage ring, there are **64** dipoles
  - The expectation value of the orbit distortion in the dipoles is **4mm**, considering a **1mrad** dipole error
  - The expection value for the orbit distortion created by the **112** focusing quadrupoles, considering an equivalent **0.1mrad** dipole error from misalignments (~ **0.1mm** for **1m** long quads of **10T/m** gradient), is **11mm**!

Transport of orbit distortion due to dipole kick

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$\begin{aligned} u_2 &= m_{11}u_1 + m_{12}u_1' \\ u_2' &= m_{21}u_1 + m_{22}u_1' \end{aligned}$$

- Consider a single dipole kick at position 1 $\delta u_1' = \frac{\delta(Bl)}{B\rho}$
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u_1' + \delta u_1') \rightarrow \delta u_2 = m_{12}\delta u_1'$$

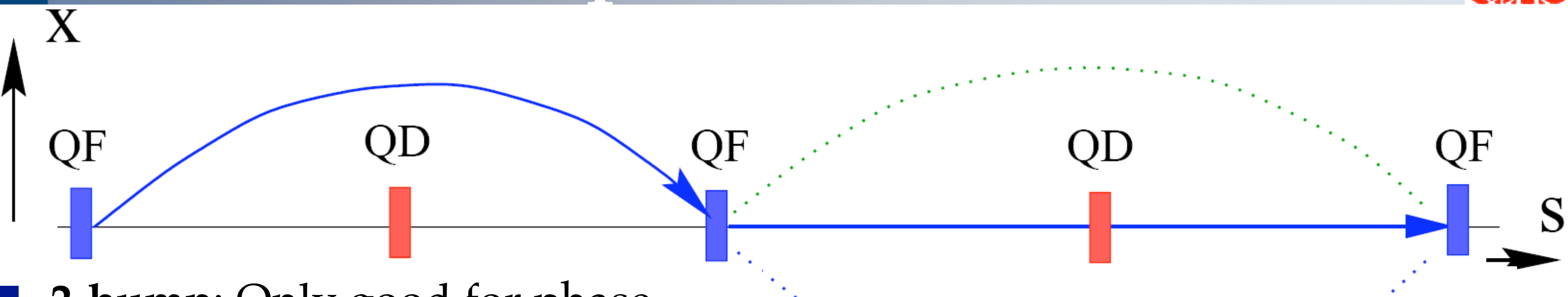
- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1 \beta_2}\sin(\phi_{12})\delta u_1'$$

Correcting the orbit distortion

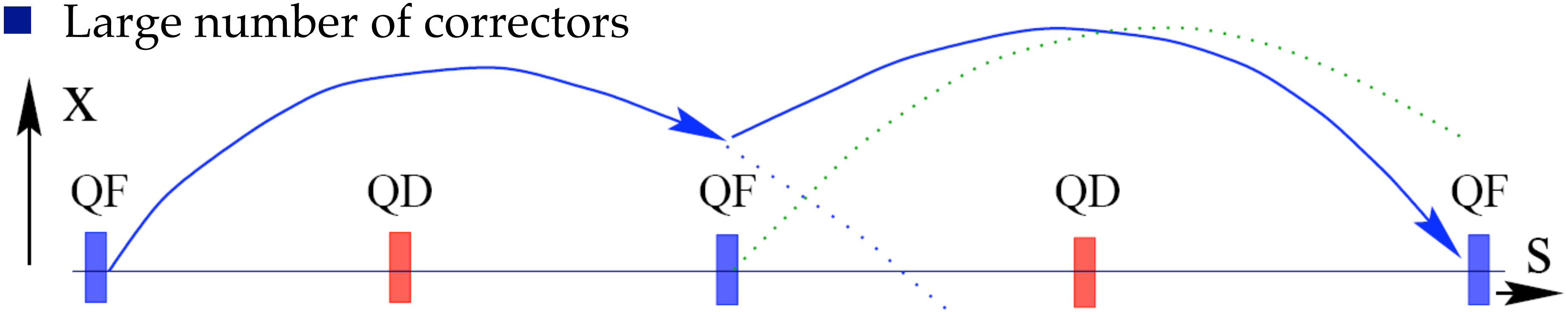
- Place horizontal and vertical dipole correctors close to the corresponding quads
- Simulate (random distribution of errors) or measure orbit in Beam position monitors (downstream of the correctors)
- Minimize orbit distortion with several methods
  - Globally
    - Harmonic , which minimizes components of the orbit frequency response after a Fourier analysis
    - Most efficient corrector (MICADO), finding the most efficient corrector for minimizing the rms orbit
    - Least square fitting
  - Locally
    - Sliding Bumps
    - Singular Value Decomposition (SVD)

Orbit bumps



- **2-bump**: Only good for phase advance equal π between correctors
- Sensitive to lattice and BPM errors
- Large number of correctors

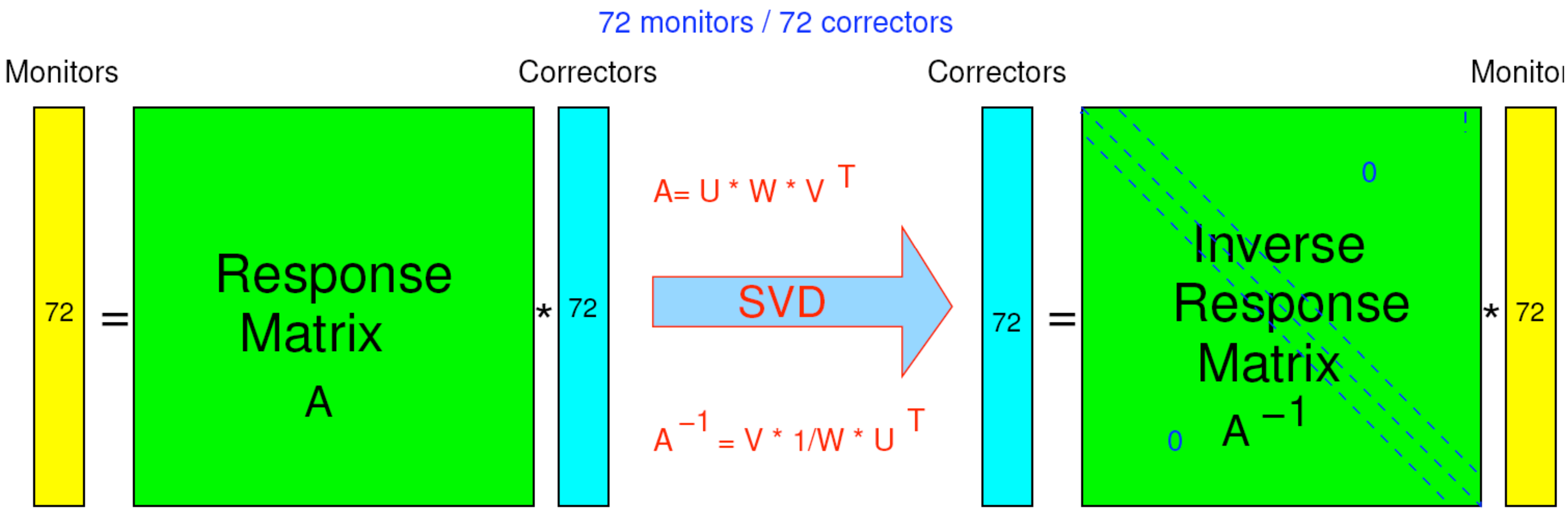
$$\delta u_2' = \frac{\sqrt{\beta_1}}{\sqrt{\beta_2}} \delta u_1'$$



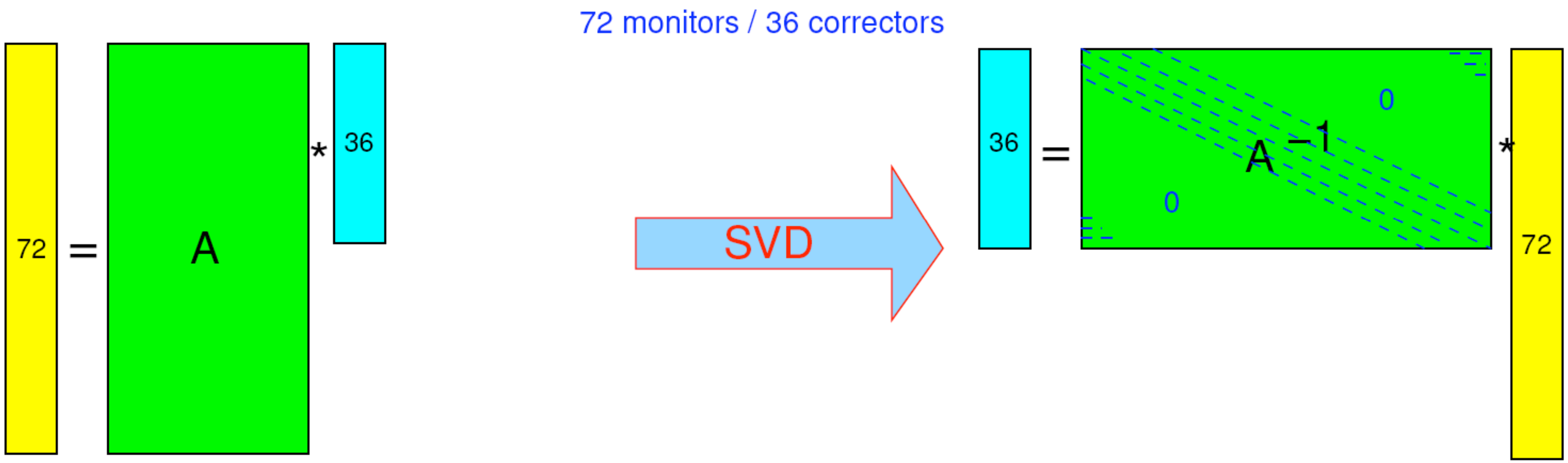
- **3-bump**: works for any lattice
- Need large number of correctors
- No control of angles

$$\frac{\sqrt{\beta_1}}{\sin \phi_{23}} \delta u_1' = \frac{\sqrt{\beta_2}}{\sin \phi_{31}} \delta u_2' = \frac{\sqrt{\beta_3}}{\sin \phi_{12}} \delta u_3'$$

Singular Value Decomposition example



=> Minimization of the RMS orbit (=0 in case of "Matrix Inversion" using all Eigenvalues)



=> Minimization of the RMS orbit (monitor averaging)

M. Boege, CAS 2003

Orbit feedback

- Closed orbit stabilization performed using slow and fast orbit feedback system.
- Slow operates every a few seconds (30 for the ESRF) and uses complete set of BPMs (~200 at the ESRF) for both planes
- Efficient in correcting distortion due to current decay in magnets or other slow processes
- Fast orbit correction system operates in a wide frequency range
- (.1Hz to 150Hz for the ESRF) correcting distortions induced by quadrupole and girder vibrations.
- Local feedback systems used to damp horizontal orbit distortion in ID straight sections, especially the ones were beam spot stabilization is critical

	β value at the BPM location	Rms motion without feedback	Rms motion with feedback	Rms motion / rms size
Horizontal	36 m	5 to 12 μm	1.2 to 2.2 μm	0.3 to 0.6 %
Vertical	5.6 m	1.5 to 2.5 μm	.8 to 1.2 μm	7 to 10 %

Gradient error and optics distortion

- Key issue for the performance -> super-periodicity preservation -> only structural resonances excited
- Broken super-periodicity -> excitations of all resonances
- Causes
  - Errors in quadrupole strengths (random and systematic)
  - Injection elements
  - Higher-order multi-pole magnets and errors
- Observables
  - Tune-shift
  - Beta-beating
  - Excitation of integer and half integer resonances

Gradient error

- Consider the transfer matrix for one turn

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a gradient error in a quad. In thin element approximation the quad matrix with and without error are

$$m_0 = \begin{pmatrix} 1 & 0 \\ -K_0(s)ds & 1 \end{pmatrix} \quad \text{and} \quad m = \begin{pmatrix} 1 & 0 \\ -(K_0(s) + \delta K)ds & 1 \end{pmatrix}$$

- The new 1-turn matrix $\mathcal{M} = m m_0^{-1} \mathcal{M}_0 = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$ which yields

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ \delta K ds(\cos(2\pi Q) - \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Consider a new matrix after 1 turn with a new $\chi = 2\pi(Q+\delta Q)$

$$\mathcal{M}^{\star} = \begin{pmatrix} \cos(\chi)+\alpha_0\sin(\chi) & \beta_0\sin(\chi) \\ -\gamma_0\sin(\chi) & \cos(\chi)-\alpha_0\sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal $\mathrm{Tra}(\mathcal{M}^{\star}) = \mathrm{Tra}(\mathcal{M})$

  which gives $2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q+\delta Q))$

- Developing the left hand side

$$\cos(2\pi(Q+\delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{2\pi\delta Q}$$

- and finally $4\pi\delta Q = \delta K ds \beta_0$
- For a quadrupole of finite length, we have

$$\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds$$

Gradient error and beta distortion

- Consider the unperturbed transfer matrix for one turn

$$\mathcal{M}_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \text{ with } A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \text{ and } \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

$$m_{12}^\star = (\beta_0 + \delta\beta)\sin(2\pi(Q + \delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

- On the other hand

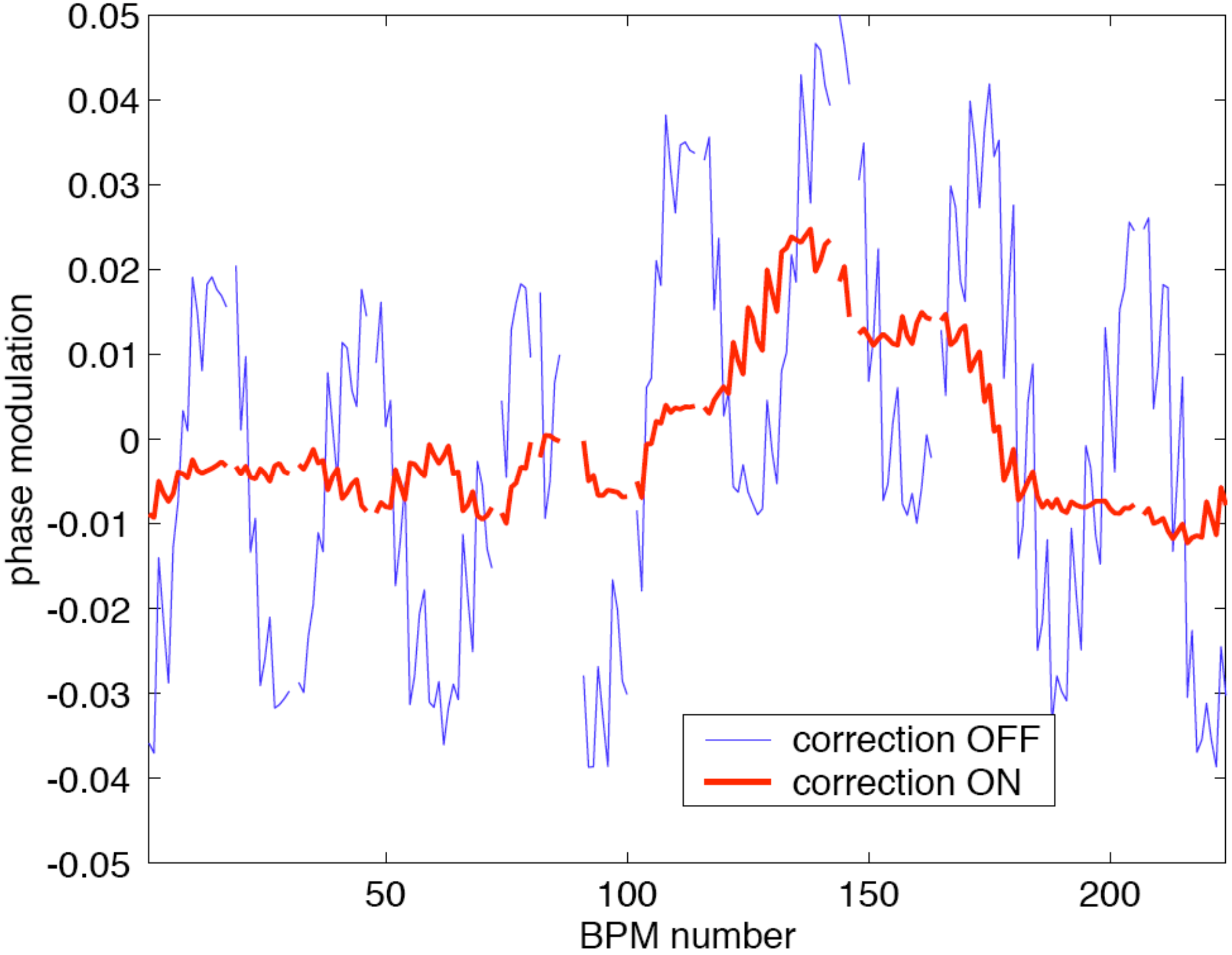
$$m_{12}^\star = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12}} - a_{12}b_{12}\delta K ds = \beta_0 \sin(2\pi Q) - a_{12}b_{12}\delta K ds$$

and $a_{12} = \sqrt{\beta_0\beta(s_1)}\sin\psi,\ b_{12} = \sqrt{\beta_0\beta(s_1)}\sin(2\pi Q - \psi)$

- Equating the two terms and integrating through the quad

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)} \int_{s_1}^{s_1+l} \beta(s)\delta K(s)\cos(2\psi - 2\pi Q) ds$$

- Consider **128** focusing arc quads in the SNS ring with **0.1%** gradient error. In this location **<β>=30m**. The length of the quads is around **1m**
- The tune-shift is



$$\delta Q = \frac{1}{4\pi} 128 \cdot 30 \frac{0.001}{20} 1 = 0.014$$

Random gradient error distribution

- For a random distribution of errors the beta variation (beating) is

$$\frac{\delta\beta}{\beta_0}_{\mathrm{rms}} = -\frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2 \beta_i^2\right)^{1/2}$$

- Optics functions beating >10% by putting random errors (0.1% of the gradient) in high dispersion quads of the ESRF storage ring
- Justifies the choice of quadrupole corrector strength

Gradient error correction

- Windings on the core of the quadrupoles or individual quadrupole correctors (TRIM)
- Simulation by introducing random distribution of quadrupole errors
- Compute the tune-shift and the optics function beta distortion
- Move working point close to integer and half integer resonance
- Minimize beta wave or quadrupole resonance width with TRIM windings
- To correct certain resonance harmonics N, strings should be powered accordingly
- Individual powering of TRIM windings can provide flexibility and beam based alignment of BPM

Linear coupling

- Betatron motion is coupled in the presence of skew quadrupoles
- The field is $(B_x, B_y) = k(x, y)$ and Hill's equations are coupled
- Motion still linear with two new eigen-mode tunes, which are always split. In the case of a thin quad: $\delta Q \propto |k|\sqrt{\beta_x \beta_y}$
- Coupling coefficients

$$|C_\pm| = \left| \frac{1}{2\pi} \oint ds k(s) \sqrt{\beta_x(s)\beta_y(s)} e^{i(\phi_x \pm \phi_y - (Q_x \pm Q_y - q_\pm) 2\pi s / C)} \right|$$

- As motion is coupled, vertical dispersion and optics function distortion appears
- Causes:
  - Random rolls in quadrupoles
  - Skew quadrupole errors
  - Off-sets in sextupoles

Vertical emittance

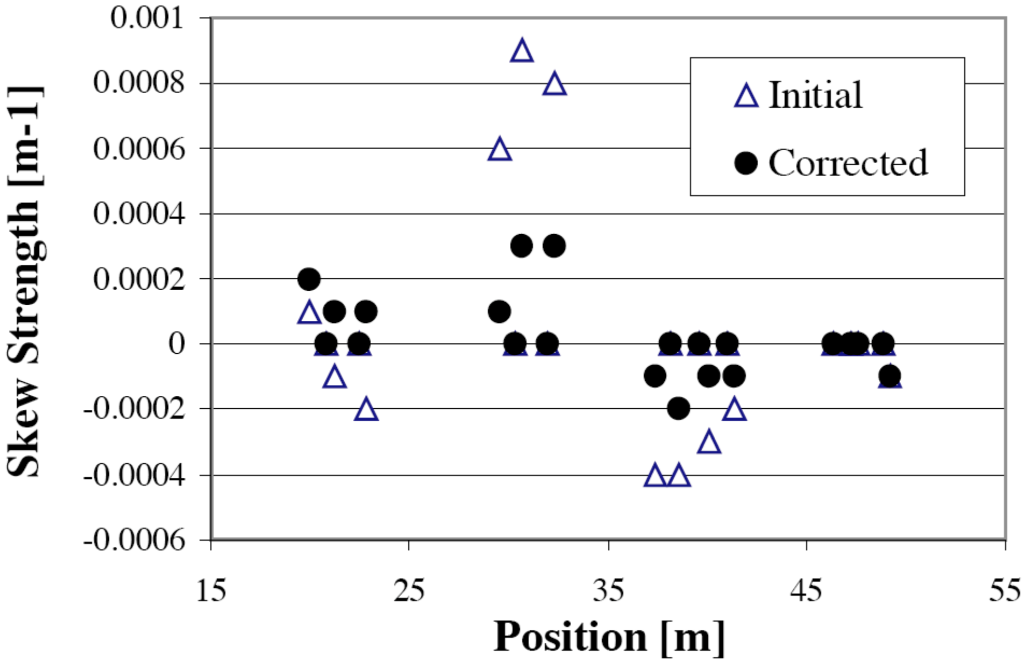
- Ideally the vertical dispersion is zero and the vertical dispersive emittance is zero, so the vertical emittance
- Real Lattice: Errors as sources of vertical emittance
  - Vertical dipoles (a_1) and skew quadrupoles (a_2)
  - Dipole and quadrupole rolls
  - Quadrupole and Sextupole vertical displacement
- Result to vertical dispersion and linear coupling needing orbit correctors and skew quadrupoles for suppression
- Emittance ratio $k_c = \frac{\epsilon_y}{\epsilon_x} \Rightarrow \epsilon_x = \frac{1}{1+k_c}\epsilon_{x0}\ , \quad \epsilon_y = \frac{k_c}{1+k_c}\epsilon_{x0}$
- Coupling corrected when coupling coefficient drops to 10^{-3} level
- Brightness proportional to the inverse of the coupling coefficient only for hard X-rays (diffraction limitation)
- Touschek lifetime proportional to $\sqrt{k_c}$

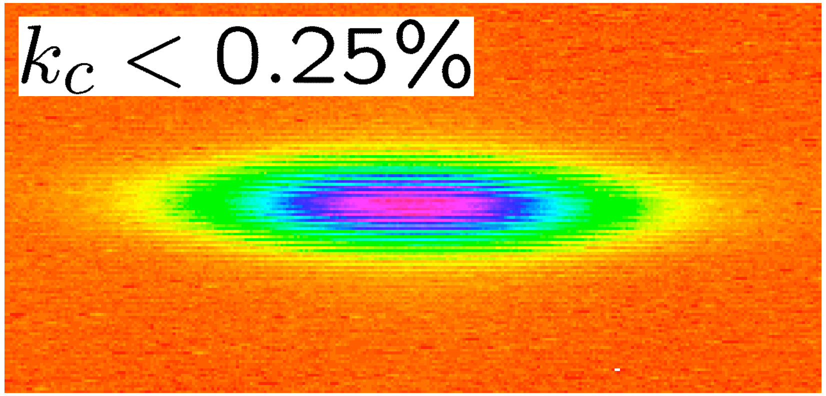
Linear coupling correction strategy

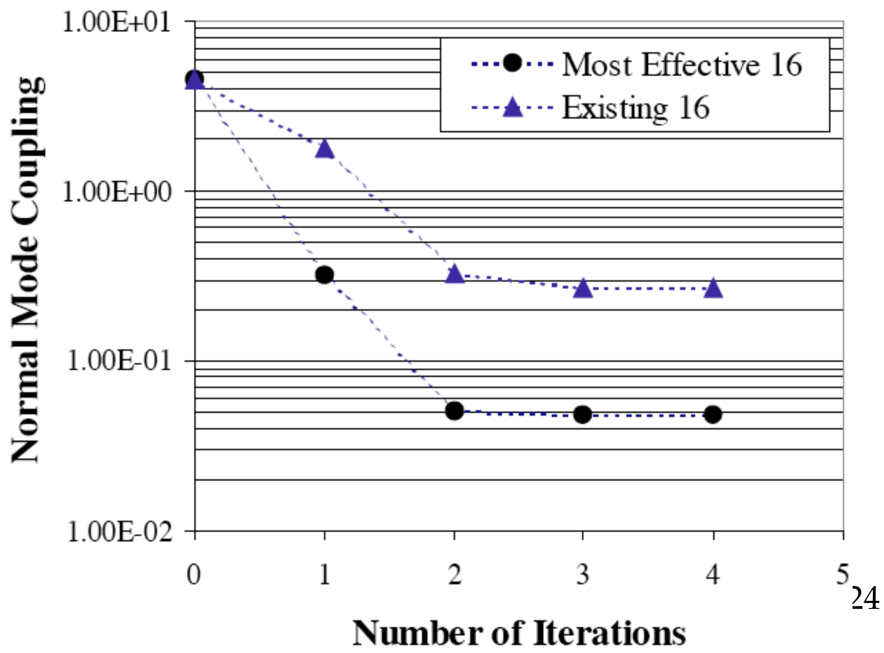
- Introduce skew quadrupole correctors
- Simulation by introducing random distribution of quadrupole errors
- Correct globally / locally coupling coefficient (or resonance driving term)
- Correct optics distortion (especially vertical dispersion)
- Move working point close to coupling resonances and repeat

Example: Coupling correction for the ESRF ring

- Local decoupling using 16 skew quadrupole correctors and coupled response matrix reconstruction
- Achieved correction of below 0.25% reaching vertical emittance of below 10pm







R. Nagaoka, EPAC 2000

Chromaticity

- Linear equations of motion depend on the energy (term proportional to dispersion)
- Chromaticity is defined as: $\xi_{x,y} = -\frac{\delta Q_{x,y}}{\delta P/P}$
- Recall that the gradient $K = \frac{G}{B\rho} = \frac{eG}{P} \rightarrow \frac{\delta K}{K} = \mp \frac{\delta P}{P}$
- This leads to dependence of tunes and optics function on energy
- For a linear lattice the tune shift is:

$$\delta Q_{x,y} = \frac{1}{4\pi} \oint \beta_{x,y} \delta K(s) ds = \frac{1}{4\pi} \frac{\delta P}{P} \oint \beta_{x,y} K(s) ds$$

- So the natural chromaticity is:

$$\xi_{x,y} = -\frac{1}{4\pi} \oint \beta_{x,y} K(s) ds$$

Example: Chromaticity in the SNS ring

- In the SNS ring, the natural chromaticity is –7.
- Consider that momentum spread $\frac{\delta P}{P} = \pm 1$
- The tune-shift for off-momentum particles is

$$\delta Q_{x,y} = \xi_{x,y}\frac{\delta P}{P} = \pm 0.07$$

- In order to correct chromaticity introduce particles which can focus off-momentum particle

↓

Sextupoles

Chromaticity from sextupoles

- The sextupole field component in the x-plane is: $B_y = \frac{S}{2}x^2$
- In an area with non-zero dispersion $x = x_0 + D\frac{\delta P}{P}$
- Than the field is $B_y = \frac{S}{2}x_0^2 + \underbrace{SD\frac{\delta P}{P}x_0}_{\text{quadrupole}} + \underbrace{\frac{S}{2}D^2\frac{\delta P}{P}^2}_{\text{dipole}}$
- Sextupoles introduce an equivalent focusing correction
$$\delta K = SD\frac{\delta P}{P}$$
- The sextupole induced chromaticity is
$$\xi_{x,y}^{S} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)S(s)D_x(s)ds$$
- The total chromaticity is the sum of the natural and sextupole induced chromaticity
$$\xi_{x,y}^{tot} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)(S(s)D_x(s) + k(s))ds$$

Chromaticity correction

- Introduce sextupoles in high-dispersion areas (not easy to find)
- Tune them to achieve desired chromaticity
- Two families are able to control horizontal and vertical chromaticity
- Sextupoles introduce non-linear fields (chaotic motion)
- Sextupoles introduce tune-shift with amplitude
- Example:
  - The ESRF ring has natural chromaticity of **-130**
  - Placing 32 sextupoles of length **0.4m** in locations where **β=30m**, and the dispersion **D=0.3m**
  - For getting **0** chromaticity, their strength should be

$$S = \frac{130 \cdot 4\pi}{30 \cdot 0.3 \cdot 32 \cdot 0.4} \approx 14\mathrm{m}^{-3}$$ or a gradient of **280 T/m²**

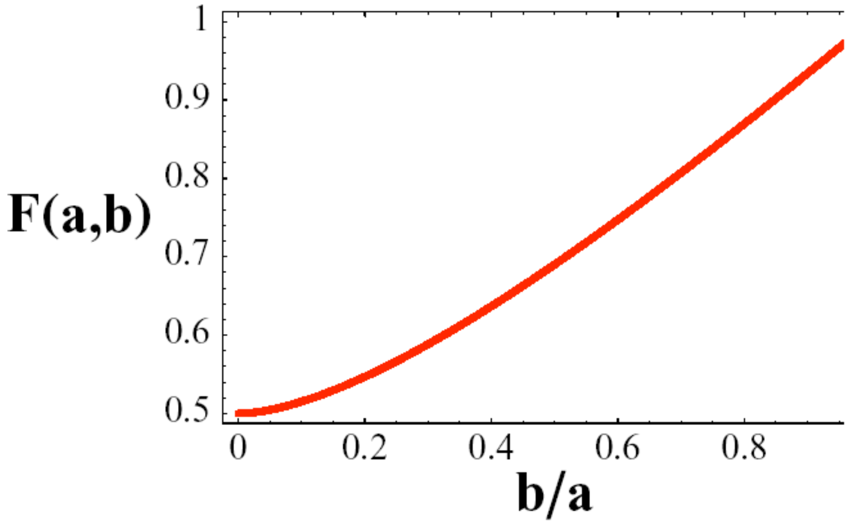
Eddy current sextupole component

$$\xi^{\text{eddy}}_{x,y}(t) = \pm\frac{1}{4\pi}\oint S^{\text{eddy}}(s,t)\eta_x(s)\beta_{x,y}(s)ds$$

Sextupole component due to Eddy currents in an elliptic vacuum chamber of a pulsing dipole

$$S^{\text{eddy}}(t) = \frac{1}{B\rho}\frac{d^2B_y}{dx^2} = \frac{1}{B\rho}\frac{\mu_0\sigma_c t\dot{B}_y}{h}F(a,b)$$

with $F(a,b) = \int_0^{\pi/2} \sin\phi\sqrt{\cos^2\phi + (b/a)^2\sin^2\phi}\, d\phi = 1/2\left[1 + \frac{b^2\text{arcsinh}(\sqrt{a^2-b^2}/b)}{a\sqrt{a^2-b^2}}\right]$



Taking into account

$$B_y(t) = \frac{B_{\max}}{1+a_E}(a_E - \cos(\omega t))$$

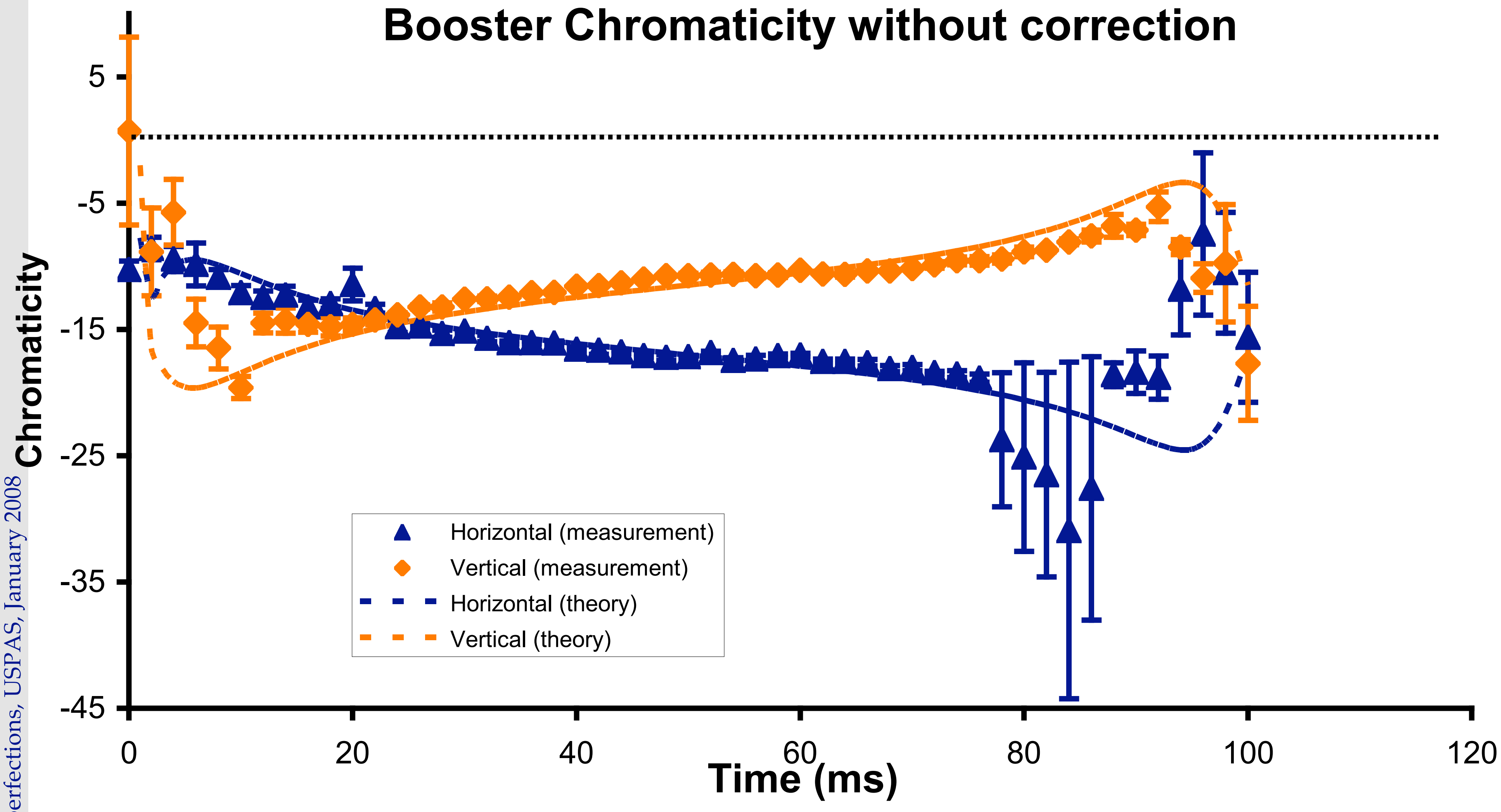
with

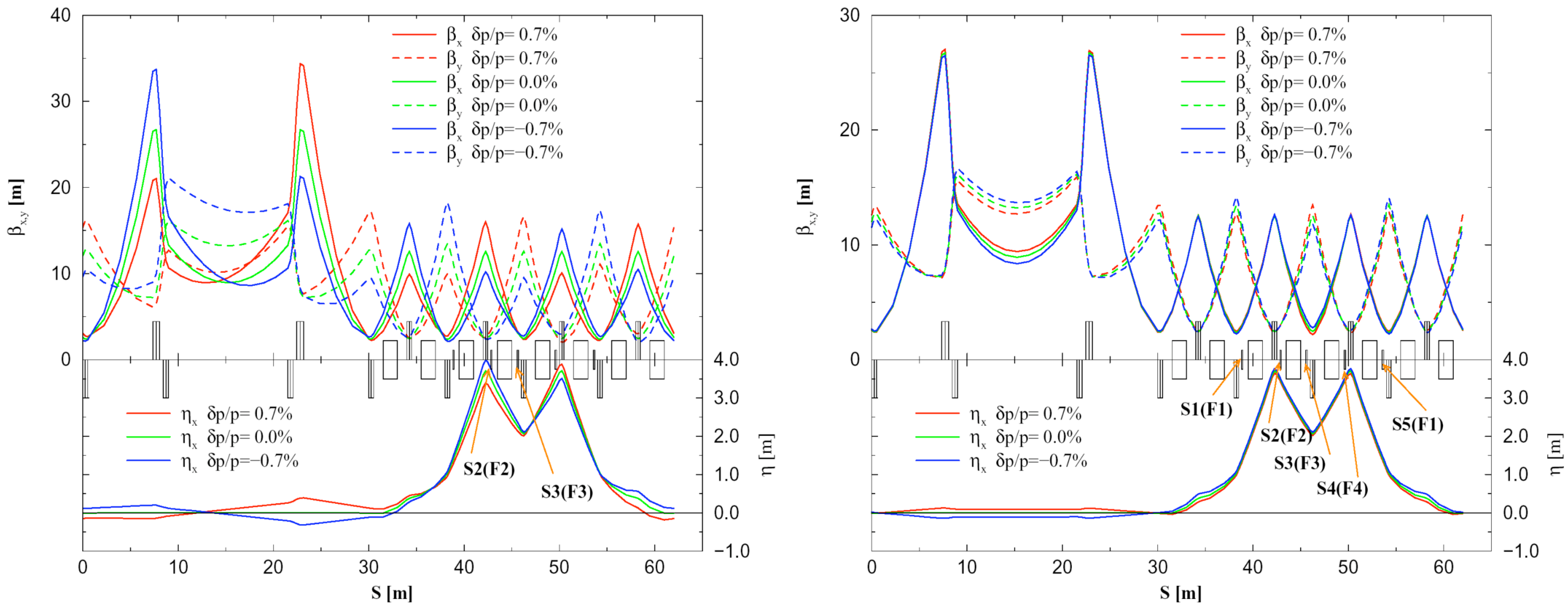
$$a_E = \frac{E_{\max}+E_{\min}}{E_{\max}-E_{\min}}$$

we get $S^{\text{eddy}}(t) = \frac{\mu_0\ \sigma_c\ t\ \omega}{h\ \rho}\frac{\sin(\omega t)}{a_E - \cos(\omega t)}F(a,b)$

Measurements vs Theory

Example: ESRF booster chromaticity





- Two families of sextupoles not enough for correcting off-momentum optics functions' distortion and second order chromaticity
- Solutions:
  - Place sextupoles accordingly to eliminate second order effects (difficult)
  - Use more families (4 in the case of of the SNS ring)
- Large optics function distortion for momentum spreads of **±0.7%**,when using only two families of sextupoles
- Absolute correction of optics beating with four families

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