Lattices for electron storage rings

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Outline

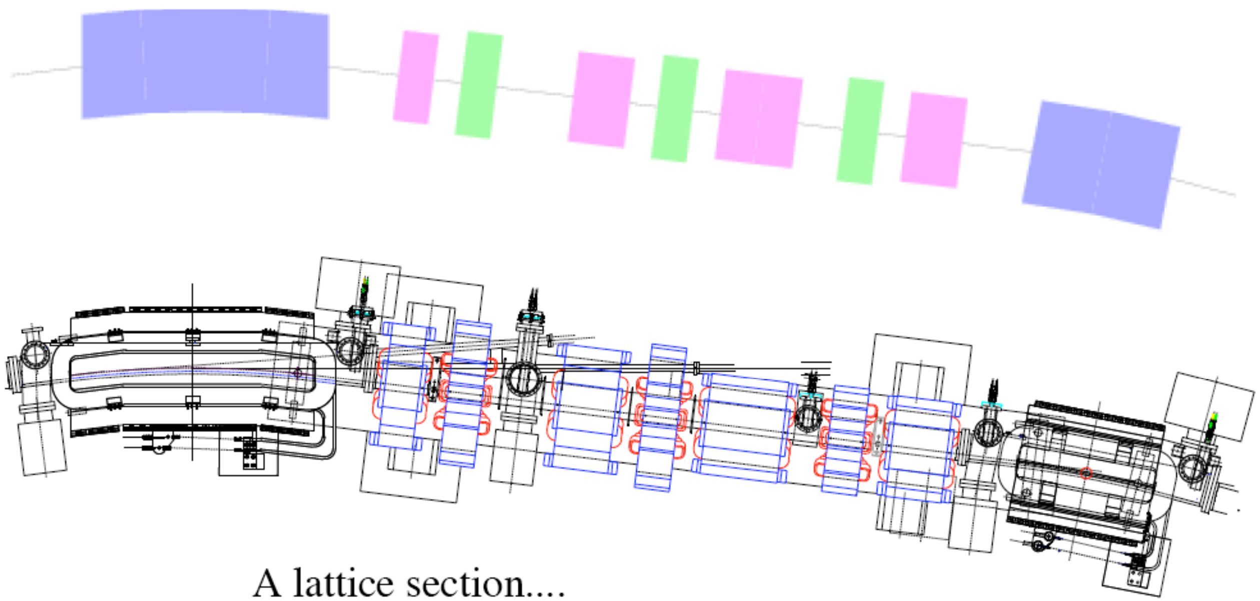
- Lattice design phases and strategy
- Building blocks, magnetic multi-pole expansion
- Reminder on matrices and betatron functions
- Low emittance lattice conditions
- Examples of low emittance lattices

Lattice design phases

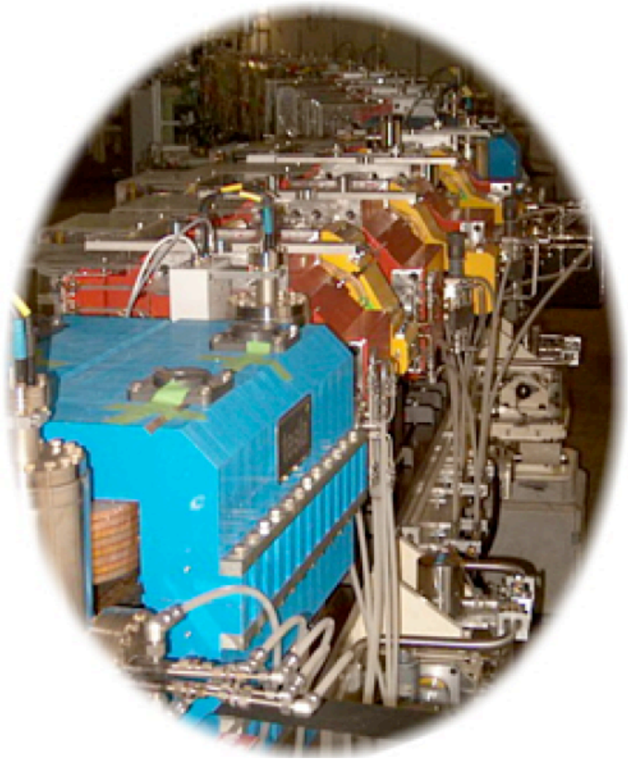
- Initial preparation
  - Performance
  - Boundary conditions and constraints
  - Building blocks (magnets)
- Linear lattice design
  - Build modules, and match them together
  - Achieve optics conditions for maximizing performance
  - Global quantities choice working point and chromaticity
- Non-linear lattice design
  - Chromaticity correction (sextupoles)
  - Dynamic aperture
- Real world
  - Include imperfections and foresee corrections

Lattice design inter-phase

Magnet Design: Technological limits, coil space, field quality
Vacuum: Impedance, pressure, physical apertures, space
Radiofrequency: Energy acceptance, bunch length, space
Diagnostics: Beam position monitors, resolution, space
Alignment: Orbit distortion and correction
Mechanical engineering: Girders, vibrations
Design engineering: Assembly, feasibility



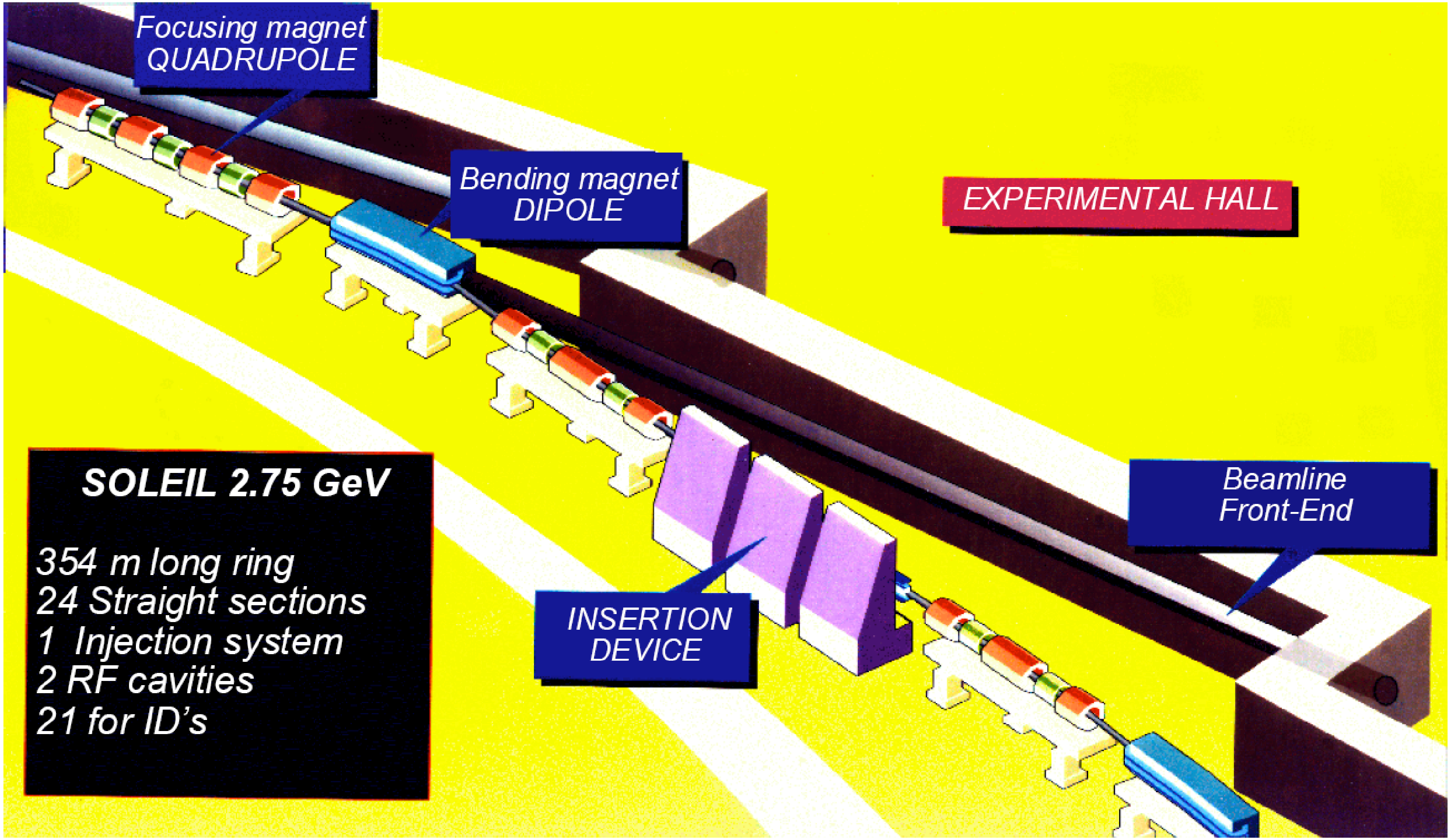
A lattice section....
(top) .....as seen by the lattice designer
(bottom) ..... as seen by the design engineer
(right) ..... and how it looks in reality



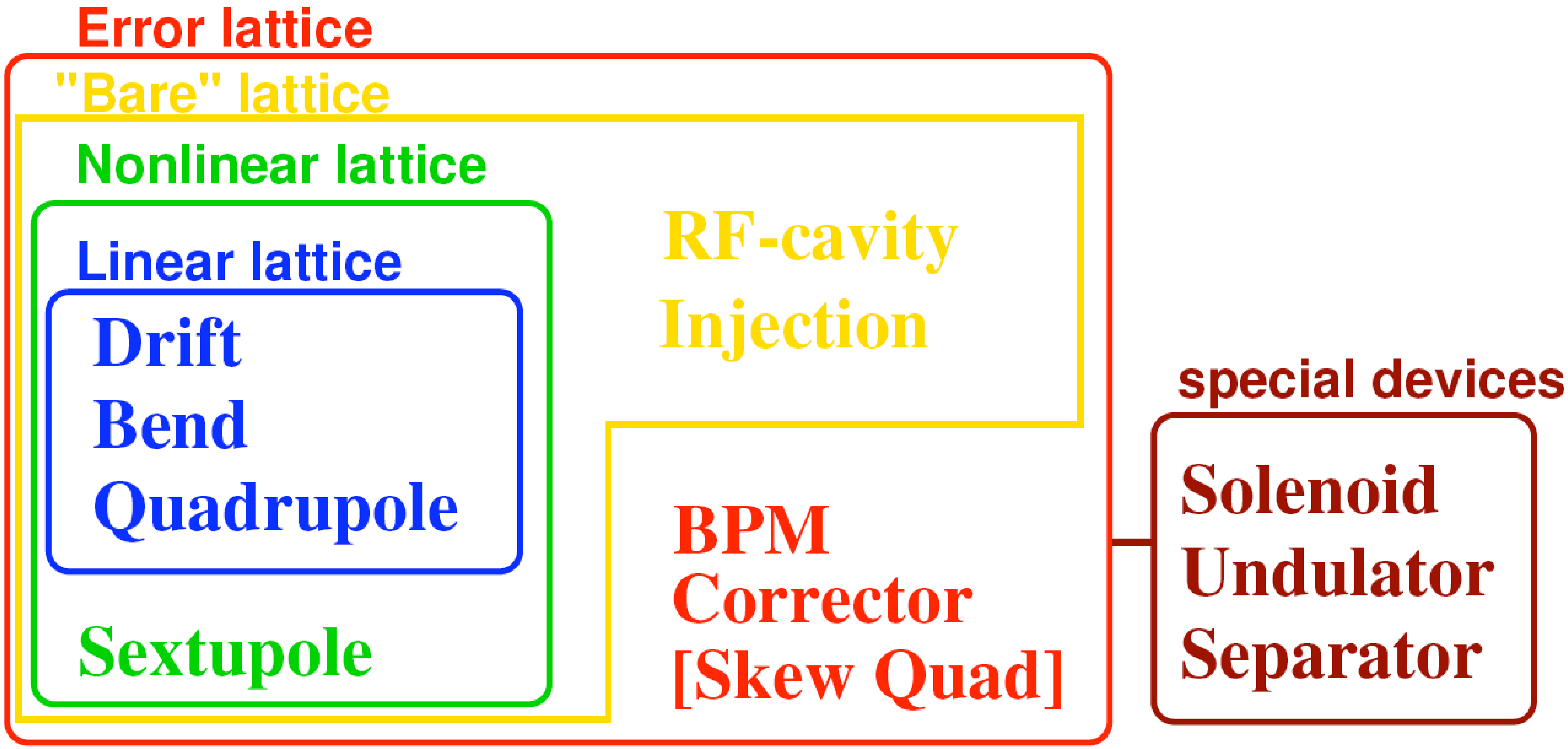
Other devices

Device	Parameter	Purpose
RF cavities	RF phase and Voltage	Acceleration, phase stability
Septum	Position and width	injection
Kicker	Integrated dipole field	injection
Orbit corrector	Integrated dipole field	Orbit correction
Quadrupole corrector	Integrated quad field	Restoring periodicity
Skew quadrupoles	Integrated skew quad field	Coupling correction
Undulators, wigglers	Number of periods, wavelength, field and gap	Synchrotron radiation
Vacuum pumps	Passive	Keep high vacuum
Beam position monitors, other instrumentation	Passive	Position, beam parameters measurement
Absorbers	Passive	Synchrotron radiation absorption

A typical lattice for a storage ring



Hierarchy of building blocks



A. Streun, CAS 2003

Magnetic multipole expansion

- From Gauss law of magnetostatics, a vector potential exist

$$\nabla \cdot \mathbf{B} = 0 \quad \rightarrow \quad \exists \mathbf{A} : \quad \mathbf{B} = \nabla \times \mathbf{A}$$

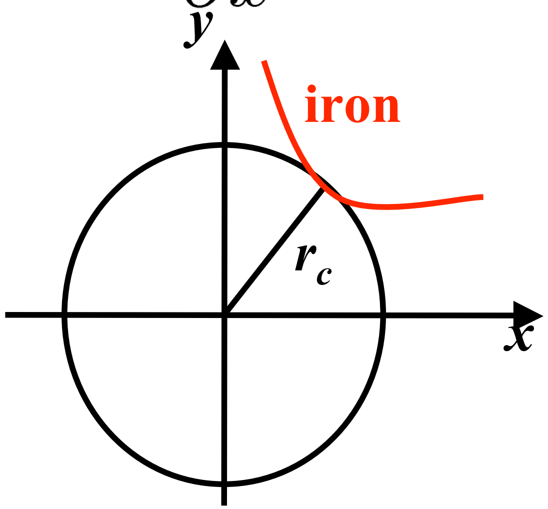
- Assuming a 2D field in x and y, the vector potential has only one component A_s. The Ampere's law in vacuum (inside the beam pipe)

$$\nabla \times \mathbf{B} = 0 \quad \rightarrow \quad \exists V : \quad \mathbf{B} = -\nabla V$$

- Using the previous equations, the relations between field components and potentials are

$$B_x = -\frac{\partial V}{\partial x} = \frac{\partial A_s}{\partial y} , \quad B_y = -\frac{\partial V}{\partial y} = -\frac{\partial A_s}{\partial x}$$

i.e. Riemann conditions of an analytic function



There exist a complex potential of $z = x+iy$ with a power series expansion convergent in a circle with radius $|z| = r_c$ (distance from iron yoke)

$$\mathcal{A}(x+iy) = A_s(x,y) + iV(x,y) = \sum_{n=1}^{\infty} \kappa_n z^n = \sum_{n=1}^{\infty} (\lambda_n + i\mu_n)(x+iy)^n$$

Magnetic multipole expansion II

- From the complex potential we can derive the fields

$$B_y + iB_x = -\frac{\partial}{\partial x}(A_s(x,y) + iV(x,y)) = -\sum_{n=1}^{\infty} n(\lambda_n + i\mu_n)(x+iy)^{n-1}$$

- Setting $b_n = -n\lambda_n$, $a_n = n\mu_n$ we have

$$B_y + iB_x = \sum_{n=1}^{\infty}(b_n - ia_n)(x+iy)^{n-1}$$

- Define normalized units $b'_n = \frac{b_n}{10^{-4}B_0}r_0^{n-1}$, $a_n = \frac{a_n}{10^{-4}B_0}r_0^{n-1}$

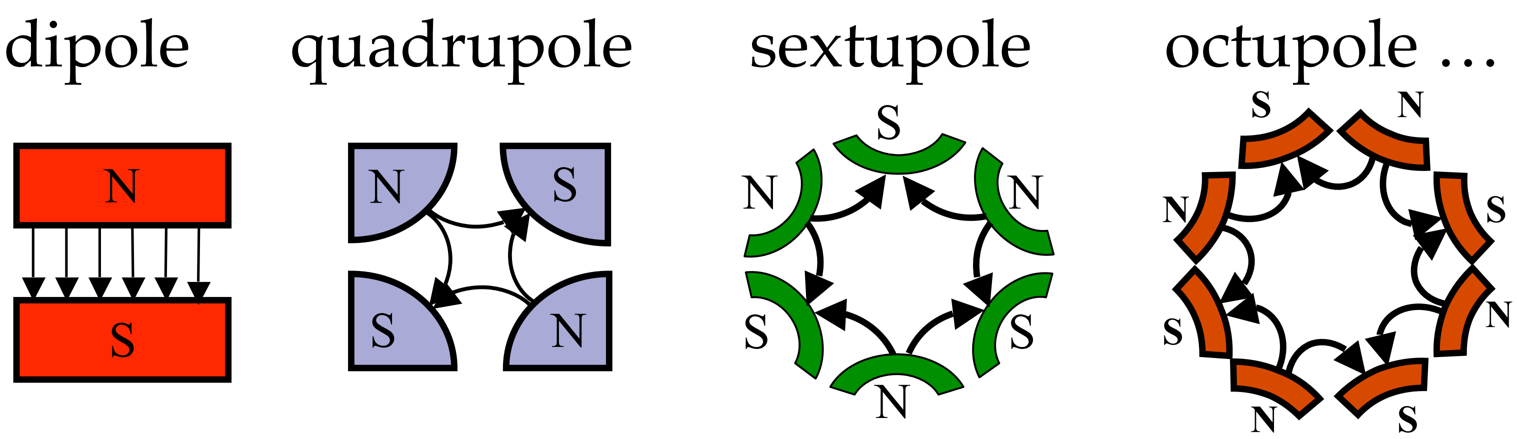
on a reference radius, 10^{-4} of the main field to get

$$B_y + iB_x = 10^{-4}B_0\sum_{n=1}^{\infty}(b'_n - ia'_n)(\frac{x+iy}{r_0})^{n-1}$$

- **Note**: ***n'=n-1*** is the US convention

Magnet definitions

- ***2n*-pole**:



n: 1 2 3 4 …

- Normal: gap appears at the horizontal plane
- Skew: rotate around beam axis by ***π/2n*** angle
- Symmetry: rotating around beam axis by ***π/n*** angle, the field is reversed (polarity flipped)

Magnetic field and aperture

	coil width $\frac{L_{tot}-L_{eff}}{2}$ [mm]	poletip field B_{pt} [T]	aperture R [mm]
Bending magnets:	65 … 150	1.5	20…35 (=g/2)
Quadrupoles:	40 … 70	0.75	30…43
Sextupoles:	40 … 80	0.6	30…50

A. Streun, CAS 2003

- Coil width should be taken into account for space considerations
- Apertures as large as necessary, as small as possible depending on acceptance imposed by lattice (a few centimeters for all main magnets)
- Current is scaled as the nth power (multi-pole order) of the radius
- Pole-tip field below 1.8T (normal conducting magnets)

Generalized transfer matrix

$$\begin{pmatrix} c_x & \frac{1}{\sqrt{K}}\, s_x & 0 & 0 & \frac{h}{K}\,(1-c_x) \\ -\sqrt{K}\, s_x & c_x & 0 & 0 & \frac{h}{\sqrt{K}}\, s_x \\ 0 & 0 & c_y & \frac{1}{\sqrt{k}}\, s_y & 0 \\ 0 & 0 & -\sqrt{k}\, s_y & c_y & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$c_x = \cos(\sqrt{K}L) \quad , \quad s_x = \sin(\sqrt{K}L) \quad , c_y = \cos(\sqrt{k}L) \quad , \quad s_y = \sin(\sqrt{k}L)$$

With $\sqrt{K} = \sqrt{\frac{1}{\rho^2} - k}$

Dipoles: $k = 0$ Quadrupoles: $\frac{1}{\rho^2} = 0$ Drifts: $\frac{1}{\rho^2} = 0\,, \quad k = 0$

Betatron motion reminder

- The linear betatron motion of a particle is described by

$$u(s) = \sqrt{\epsilon\beta(s)}\cos(\psi(s)+\psi_0) + D(s)\frac{\Delta P}{P} \qquad \text{and}$$

$$u'(s) = \sqrt{\frac{\epsilon}{\beta(s)}}\left(\sin(\psi(s)+\psi_0) + \alpha(s)\cos(\psi(s)+\psi_0)\right) + D'(s)\frac{\Delta P}{P}$$

with $\alpha,\ \beta,\ \gamma$ the twiss functions $\alpha(s) = -\frac{\beta(s)'}{2},\quad \gamma = \frac{1+\alpha(s)^2}{\beta(s)}$

ψ the betatron phase $\psi(s) = \int \frac{ds}{\beta(s)}$

The beta function defines the envelope (machine aperture)

$$E(s) = \sqrt{\epsilon\beta(s)}$$

Twiss parameters evolve as

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{s_2} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & m_{11}m_{22}+m_{12}m_{21} & -m_{22}m_{12} \\ m_{21}^2 & 2m_{22}m_{21} & m_{22}^2 \end{pmatrix}\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{s_1}$$

Lattice section transfer matrix

- Generalized transfer matrix $\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \mathcal{M}_{0\to s} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$

$$\mathcal{M}_{0\to s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\psi + \alpha_0 \sin\Delta\psi) & \sqrt{\beta(s)\beta_0}\sin\Delta\psi \\ \frac{(a_0 - a(s))\cos\Delta\psi - (1+\alpha_0\alpha(s))\sin\Delta\psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}(\cos\Delta\psi - \alpha_0\sin\Delta\psi) \end{pmatrix}$$

- Periodic cell

$$\mathcal{M}_C = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- Mirror symmetric cell

$$\mathcal{M}_C = \begin{pmatrix} \cos\mu & \beta\sin\mu \\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix}$$

LIGHT SOURCE PERFORMANCE

High Brilliance

$$\tilde{B} \propto \frac{I}{\epsilon_x \epsilon_y}$$

Current

RF cavities design

Emittance

Lattice design

Equilibrium emittance reminder

$$\epsilon_x = \frac{C_q \gamma^2}{\mathcal{J}_x} \frac{\oint \frac{\mathcal{H}_x(s)}{|\rho_x|^3} ds}{\oint \frac{1}{\rho_x^2} ds} \qquad C_q = \frac{55}{32\sqrt{3}} \frac{h}{m_0 c} = 3.83 \times 10^{-13} \text{ m}$$

with the dispersion emittance defined as

$$\mathcal{H}(s) = \beta(s)\eta(s)'^2 + 2\alpha(s)\eta(s)\eta'(s) + \gamma(s)\eta(s)^2$$

- For isomagnetic ring with separated function magnets the equilibrium emittance is written

$$\epsilon_x = 1470 \frac{E^2}{\rho} \frac{1}{l_{\text{bend}}} \int_0^{l_{\text{bend}}} \mathcal{H}_x(s) ds$$

- Smaller bending angle and lower energy reduce emittance

Twiss functions through a dipole

- Consider the transport matrix of a bending magnet (ignoring edge focusing)

$$\mathcal{M}_{\rm sector} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

- Consider at its entrance the initial optics functions $\beta 0, \alpha 0, \gamma 0, \eta 0, \eta 0'$
- The evolution of the twiss functions, dispersion and dispersion derivative are given by

$$\begin{pmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}\left[\frac{s}{\rho}\right]^2 & -\rho\,\mathrm{Sin}\left[\frac{2s}{\rho}\right] & \rho^2\,\mathrm{Sin}\left[\frac{s}{\rho}\right]^2 \\ \frac{\mathrm{Sin}\left[\frac{2s}{\rho}\right]}{2\rho} & \mathrm{Cos}\left[\frac{2s}{\rho}\right] & -\frac{1}{2}\rho\,\mathrm{Sin}\left[\frac{2s}{\rho}\right] \\ \frac{\mathrm{Sin}\left[\frac{s}{\rho}\right]^2}{\rho^2} & \frac{\mathrm{Sin}\left[\frac{2s}{\rho}\right]}{\rho} & \mathrm{Cos}\left[\frac{s}{\rho}\right]^2 \end{pmatrix} \begin{pmatrix} \beta(0) \\ \alpha(0) \\ \gamma(0) \end{pmatrix}$$

$$\begin{aligned} \eta(s) &= \eta_0\cos\left(\frac{s}{\rho}\right) + \eta_0'\rho\sin\left(\frac{s}{\rho}\right) + \rho\left(1-\cos\left(\frac{s}{\rho}\right)\right) \\ \eta'(s) &= -\frac{\eta_0}{\rho}\sin\left(\frac{s}{\rho}\right) + \eta_0'\cos\left(\frac{s}{\rho}\right) + \sin\left(\frac{s}{\rho}\right) \end{aligned}$$

Average dispersion emittance

- The dispersion emittance through he dipole is written as

$$\mathcal{H}(s) =$$

$$\gamma 0 \left(\eta 0^2 + \frac{1}{2}\,\eta 0 \left(-4\,\rho + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\right) + \frac{1}{2}\left(3\,\rho^2 - 4\,\rho^2\,\mathrm{Cos}\left[\frac{s}{\rho}\right] + \rho^2\,\mathrm{Cos}\left[\frac{2\,s}{\rho}\right]\right)\right) +$$

$$\beta 0 \left(\frac{1}{2}\left(1 - \mathrm{Cos}\left[\frac{2\,s}{\rho}\right]\right) + 2\,\mathrm{Sin}\left[\frac{s}{\rho}\right]\eta 0' + (\eta 0')^2\right) +$$

$$\alpha 0 \left(\frac{1}{2}\left(-4\,\rho\,\mathrm{Sin}\left[\frac{s}{\rho}\right] + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\mathrm{Sin}\left[\frac{s}{\rho}\right]\right) + \frac{1}{2}\left(-4\,\rho + 4\,\rho\,\mathrm{Cos}\left[\frac{s}{\rho}\right]\right)\eta 0' + \eta 0\left(2\,\mathrm{Sin}\left[\frac{s}{\rho}\right] + 2\,\eta 0'\right)\right)$$

and its average along the dipole of length *l*

$$\langle \mathcal{H}(s) \rangle =$$

$$\gamma 0 \left(\eta 0^2 - \frac{2\,\eta 0\,\rho\left(l - \rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right]\right)}{l} + \frac{\rho^2\left(6\,l - 8\,\rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right] + \rho\,\mathrm{Sin}\left[\frac{2\,l}{\rho}\right]\right)}{4\,l}\right) +$$

$$\beta 0 \left(\frac{1}{2} - \frac{\rho\,\mathrm{Sin}\left[\frac{2\,l}{\rho}\right]}{4\,l} - \frac{2\,\rho\left(-1 + \mathrm{Cos}\left[\frac{l}{\rho}\right]\right)\eta 0'}{l} + (\eta 0')^2\right) +$$

$$\alpha 0 \left(-\frac{4\,\rho^2\,\mathrm{Sin}\left[\frac{l}{2\,\rho}\right]^4}{l} - \frac{2\,\rho\left(l - \rho\,\mathrm{Sin}\left[\frac{l}{\rho}\right]\right)\eta 0'}{l} + \frac{2\,\eta 0\left(\rho - \rho\,\mathrm{Cos}\left[\frac{l}{\rho}\right] + l\,\eta 0'\right)}{l}\right)$$

Optics functions for minimum emittance

- Take the derivative of the dispersion emittance with respect to the initial optics functions and equate it to zero to find the minimum conditions
- Non-zero dispersion (general case)

$$\beta_0 = \frac{\rho^3\ (2\ (-1+\theta^2+\mathrm{Cos}[2\,\theta])+\theta\,\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{\theta\,\rho^4\ (-9\,\theta+2\,\theta^3+8\,\theta\,\mathrm{Cos}[\theta]+\theta\,\mathrm{Cos}[2\,\theta]+8\,\mathrm{Sin}[\theta]-4\,\mathrm{Sin}[2\,\theta])}}$$

$$\alpha_0 = \frac{\rho^2\ (-\theta+\theta\,\mathrm{Cos}[2\,\theta]+4\,\mathrm{Sin}[\theta]-2\,\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{\theta\,\rho^4\ (-9\,\theta+2\,\theta^3+8\,\theta\,\mathrm{Cos}[\theta]+\theta\,\mathrm{Cos}[2\,\theta]+8\,\mathrm{Sin}[\theta]-4\,\mathrm{Sin}[2\,\theta])}}$$

$$\eta_0 = \rho - \frac{\rho\,\mathrm{Sin}[\theta]}{\theta} \quad \text{and} \quad \eta_0' = \frac{-1+\mathrm{Cos}[\theta]}{\theta}$$

- Zero dispersion (and its derivative)

$$\beta_0 = \frac{\rho^2\ (6\,\theta-8\,\mathrm{Sin}[\theta]+\mathrm{Sin}[2\,\theta])}{\sqrt{2}\sqrt{-\rho^2\ (9-6\,\theta^2-16\,\mathrm{Cos}[\theta]+7\,\mathrm{Cos}[2\,\theta]+8\,\theta\,\mathrm{Sin}[\theta]+2\,\theta\,\mathrm{Sin}[2\,\theta])}}$$

$$\alpha_0 = \frac{4\,\rho\,\mathrm{Sin}\left[\frac{\theta}{2}\right]^4}{\sqrt{-\frac{9\,\rho^2}{2}+3\,\theta^2\,\rho^2-\frac{1}{2}\,\rho\ (-16\,\rho\,\mathrm{Cos}[\theta]+7\,\rho\,\mathrm{Cos}[2\,\theta]+2\,\theta\,\rho\ (4\,\mathrm{Sin}[\theta]+\mathrm{Sin}[2\,\theta]))}}$$

Minimum emittance conditions

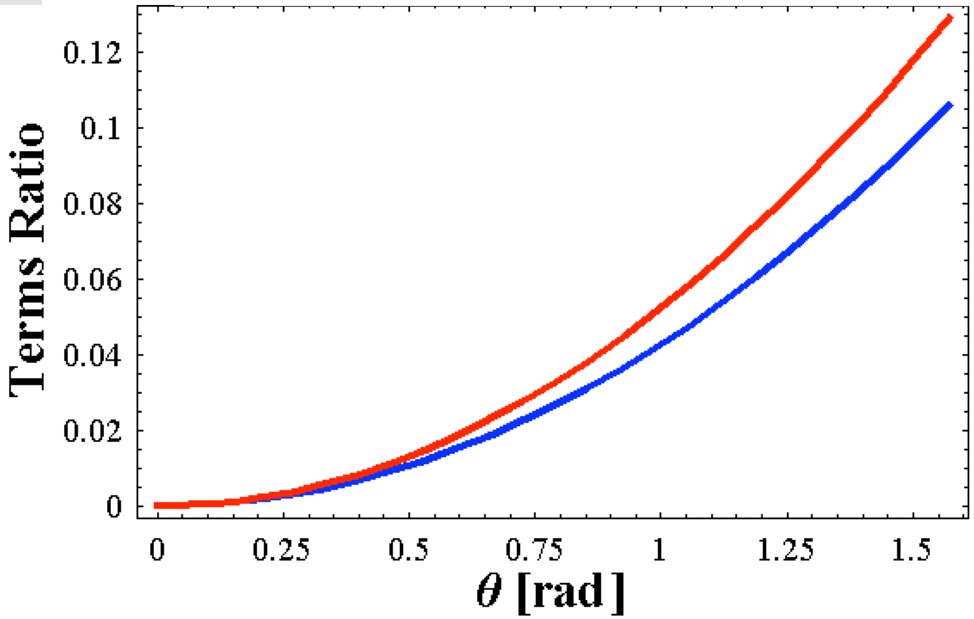
- In the general case, the equilibrium emittance takes the form

$$\epsilon_x = \frac{1}{\mathrm{Jx}\,\theta^2}\left(735\sqrt{2}\,\mathrm{En}^2\sqrt{\theta\,(-9\,\theta + 2\,\theta^3 + 8\,\theta\,\mathrm{Cos}[\theta] + \theta\,\mathrm{Cos}[2\,\theta] + 8\,\mathrm{Sin}[\theta] - 4\,\mathrm{Sin}[2\,\theta])}\right)$$

and expanding on θ we have
$$\epsilon_x = \frac{49\sqrt{\frac{5}{3}}\,\mathrm{En}^2\,\theta^3}{2\,\mathrm{Jx}} - \frac{7\left(\sqrt{\frac{3}{5}}\,\mathrm{En}^2\right)\theta^5}{4\,\mathrm{Jx}} + O[\theta]^6$$

- In the 0-dispersion case,

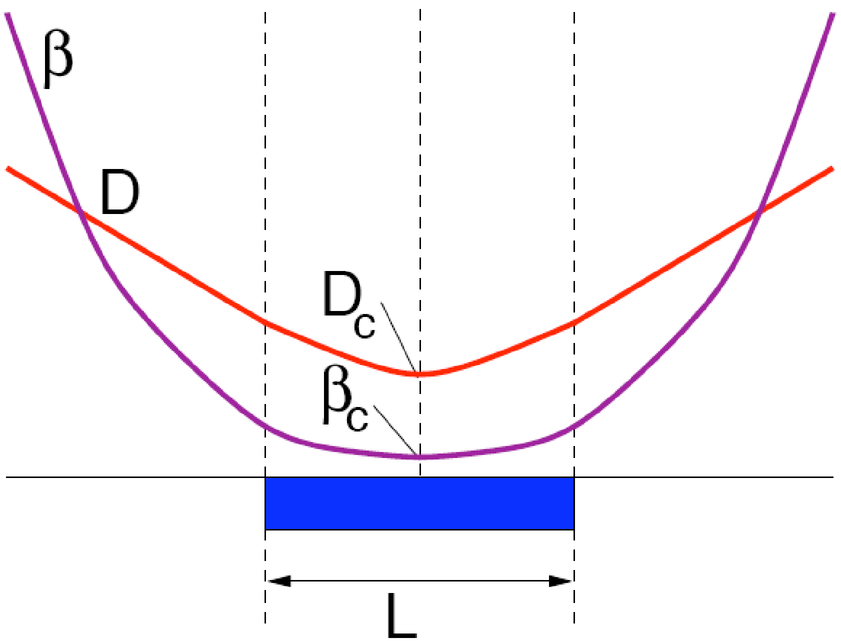
$$\epsilon_x = \frac{735\sqrt{2}\,\mathrm{En}^2\sqrt{-9 + 6\,\theta^2 + 16\,\mathrm{Cos}[\theta] - 7\,\mathrm{Cos}[2\,\theta] - 4\,\theta\,(2 + \mathrm{Cos}[\theta])\,\mathrm{Sin}[\theta]}}{\mathrm{Jx}\,\theta}$$

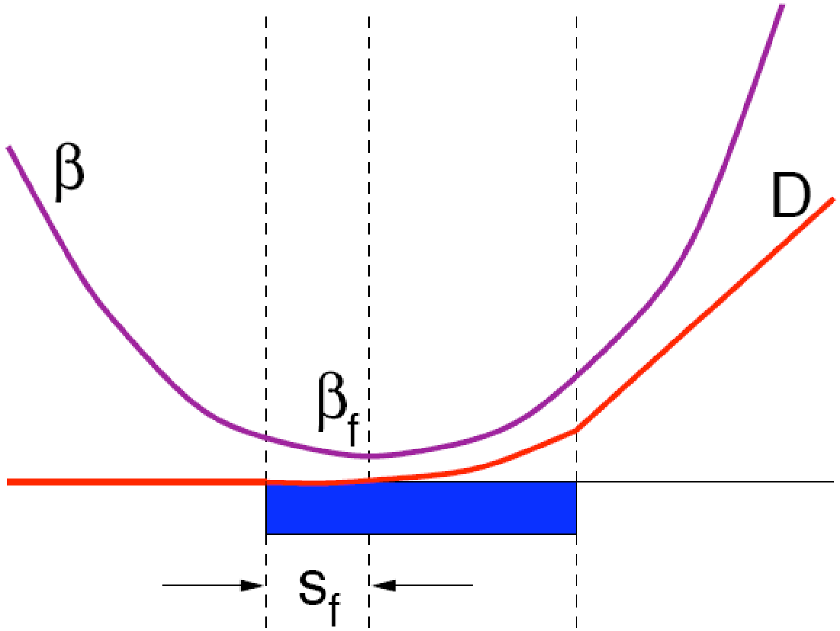


$$\epsilon_x = \frac{49\sqrt{15}\,\mathrm{En}^2\,\theta^3}{2\,\mathrm{Jx}} - \frac{77\,\mathrm{En}^2\,\theta^5}{4\left(\sqrt{15}\,\mathrm{Jx}\right)} + O[\theta]^6$$

- The second order term is negligible (less the 1% for θ < 20 deg.)
- Note that in both cases the emittance depends on the 3rd power of the bending angle
- The emittance for non-zero dispersion is 3 times smaller

Optics functions





$$\beta_0 = \frac{8\,\rho\,\theta}{\sqrt{15}} - \frac{4\,\rho\,\theta^3}{5\sqrt{15}} + \frac{257\,\rho\,\theta^5}{7350\sqrt{15}} + O[\theta]^6$$

$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{5}{3}}\,\theta^2 + \frac{269\,\theta^4}{1470\sqrt{15}} + O[\theta]^6$$

$$\eta_0 = \frac{\rho\,\theta^2}{6} - \frac{\rho\,\theta^4}{120} + O[\theta]^6$$

$$\eta'_0 = -\frac{\theta}{2} + \frac{\theta^3}{24} - \frac{\theta^5}{720} + O[\theta]^6$$

$$\beta_0 = 2\sqrt{\frac{3}{5}}\,\rho\,\theta - \frac{2\,\rho\,\theta^3}{5\sqrt{15}} + \frac{11\sqrt{\frac{3}{5}}\,\rho\,\theta^5}{2450} + O[\theta]^6$$

$$\alpha_0 = \sqrt{15} - \frac{4}{7}\sqrt{\frac{3}{5}}\,\theta^2 + \frac{13}{490}\sqrt{\frac{3}{5}}\,\theta^4 + O[\theta]^6$$

$$\eta_0 = \eta'_0 = 0$$

Deviation from the minimum emittance

- Introduce the dimensionless quantities $\bar{\beta}_0 = \frac{\beta_0}{\beta_{0;\min}}$

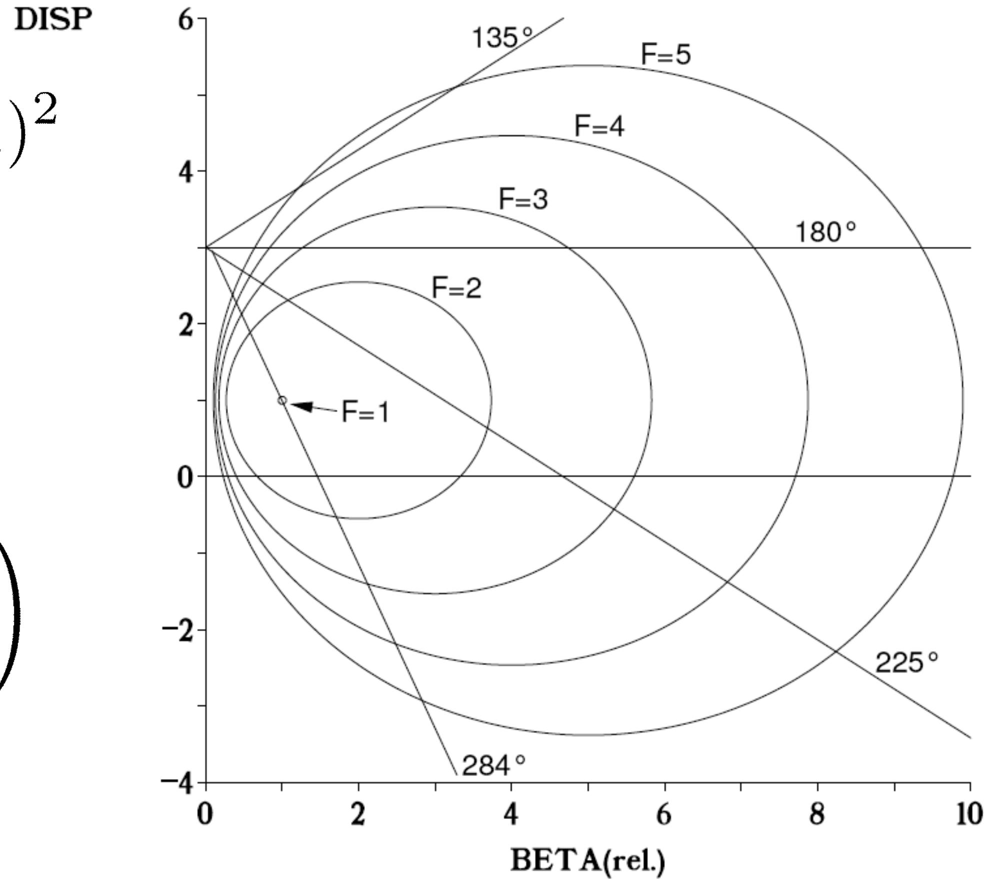
$$\bar{\eta}_0 = \frac{\eta_0}{\eta_{0;\min}} \text{ and } F = \frac{\epsilon_x}{\epsilon_{x;\min}}$$

- Introduce them into the expression of the mean dispersion emittance to get

$$F^2 = (\bar{\beta}_0 - F)^2 + 5/4(\bar{\eta}_0 - 1)^2$$

- The curves of equal relative emittance are ellipses
- The phase advance for a mirror symmetric cell is

$$\mu = \arctan\left(\frac{6}{\sqrt{15}}\frac{\bar{\beta}_0}{\bar{\eta}_0 - 3}\right)$$



- The optimum phase advance for reaching the absolute minimum emittance (F=1) is unique (284.5°)!

Effective emittance

Horizontal dispersion in the straight section $\eta_x \neq 0$

Enlargement of the beam size through the electron energy spread at the ID

Reaching the minimum theoretical emittance

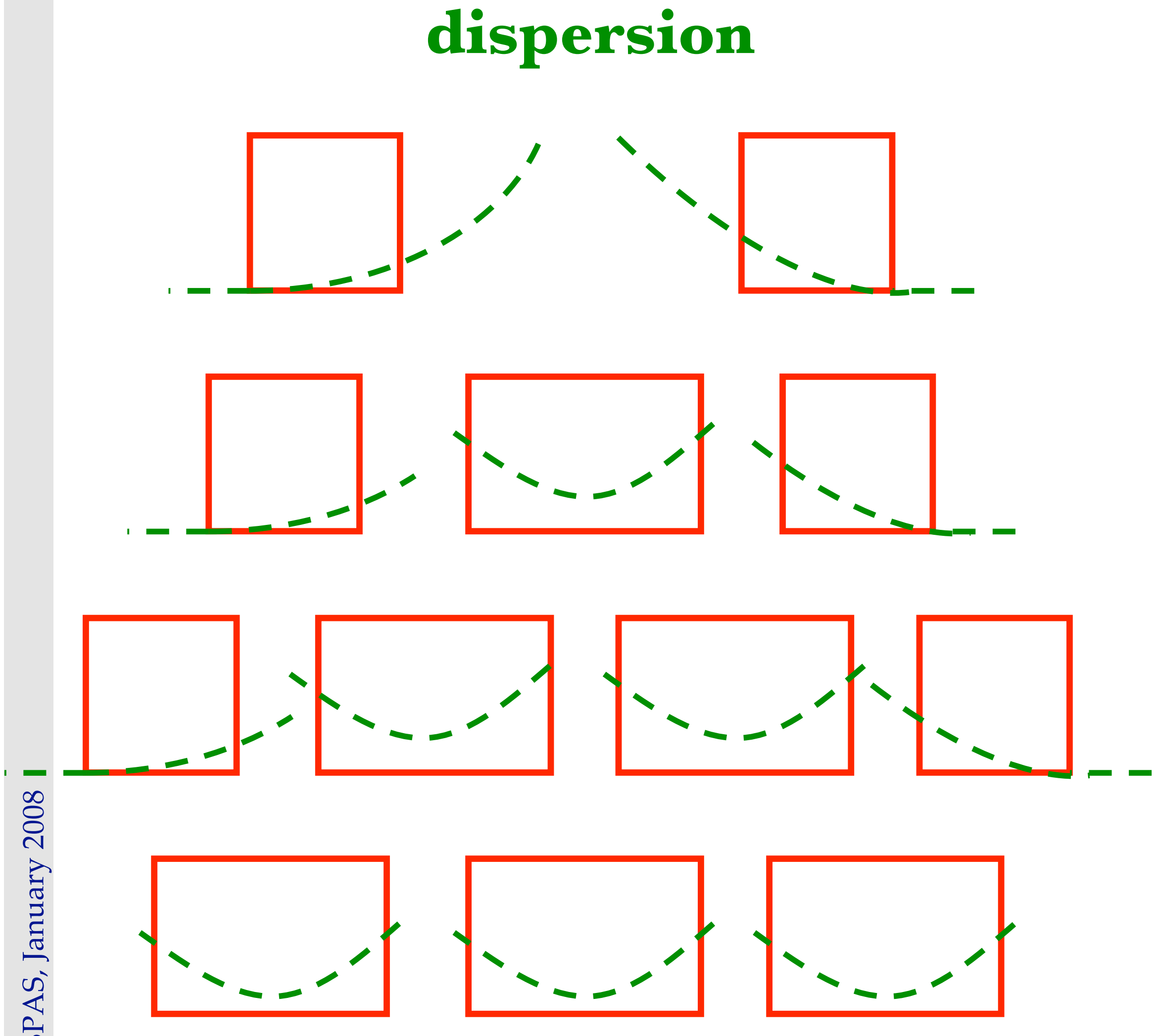
The brilliance $\tilde{B} \propto \dfrac{I}{\epsilon_{x_{eff}}(s_{ID})\epsilon_{y_{eff}}(s_{ID})}$ is inversely proportional to

the **effective emittance** $\epsilon_{x;eff}(s)^2 \equiv \langle x(s)^2\rangle\langle x'(s)^2\rangle - \langle x(s)x'(s)\rangle^2$.

After replacing the expressions for position and angles and consider that the alpha function and dispersion derivative are zero on the ID

$$\boxed{\epsilon_{x_{eff}}(s_{ID}) = \sqrt{\epsilon_x^2 + \mathcal{H}_x(s_0)\epsilon_x\sigma_\delta^2}}$$

Low emittance lattices



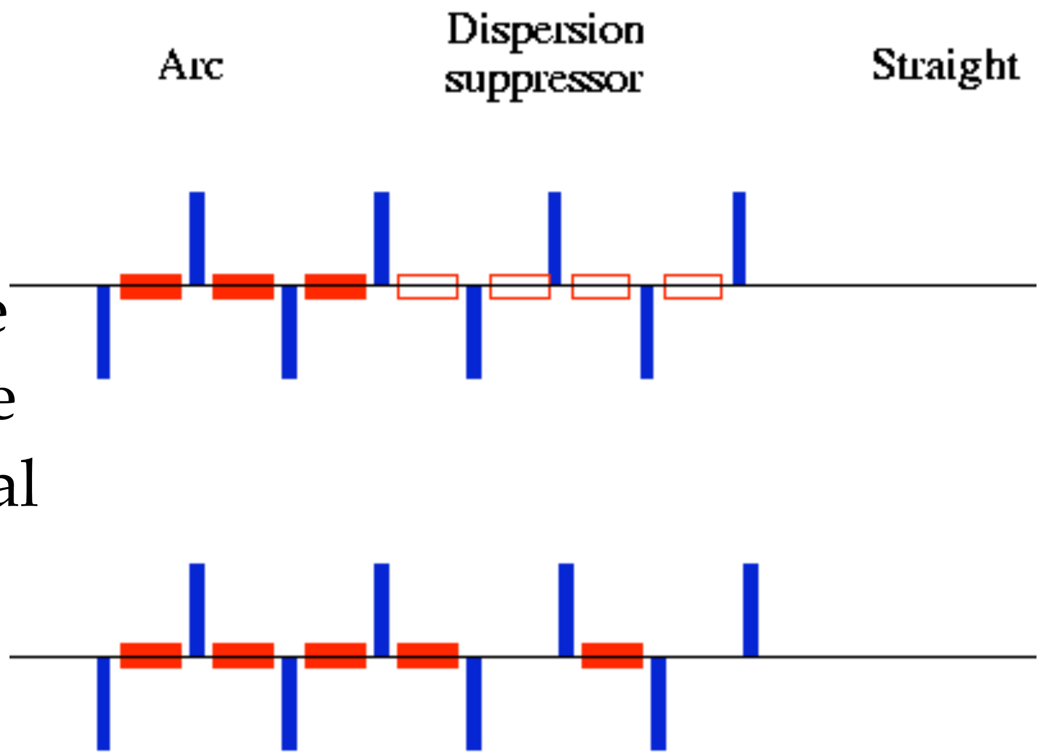
- Double Bend Achromat (DBA)
- Triple Bend Achromat (TBA)
- Quadruple Bend Achromat (QBA)
- Minimum Emittance Lattice (MEL)

Dispersion suppressors

- Dispersion has to be eliminated in special areas like injection, extraction or interaction points (orbit independent to momentum spread)
- Use **dispersion suppressors**
  - Eliminate two dipoles in a FODO cell (missing dipole)
  - Set last dipoles with different bending angles

$$
\begin{aligned}
\theta_1 &= \theta\left(1 - \frac{1}{4\sin^2\mu_{\mathrm{HFODO}}}\right) \\
\theta_2 &= \frac{\theta}{4\sin^2\mu_{\mathrm{HFODO}}}
\end{aligned}
$$

  - For equal bending angle dipoles the FODO phase advance should be equal to **π/2**



Chasmann-Green cell

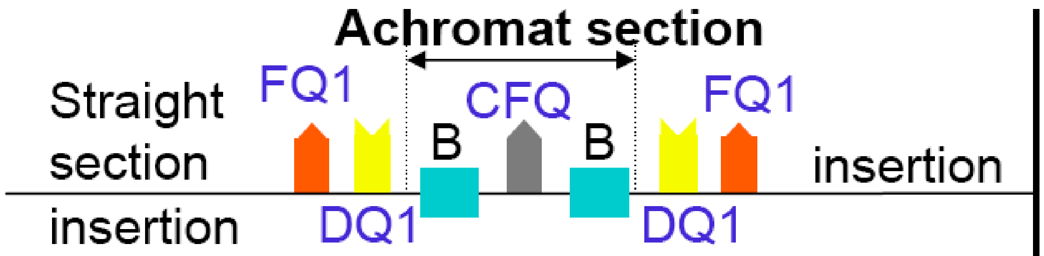
- Double bend achromat with unique central quadrupole
- Achromatic condition is assured by tuning the central quadrupole
- Minimum emittance with a quadrupole doublet in either side of the bends
- The required focal length of the quad is given by

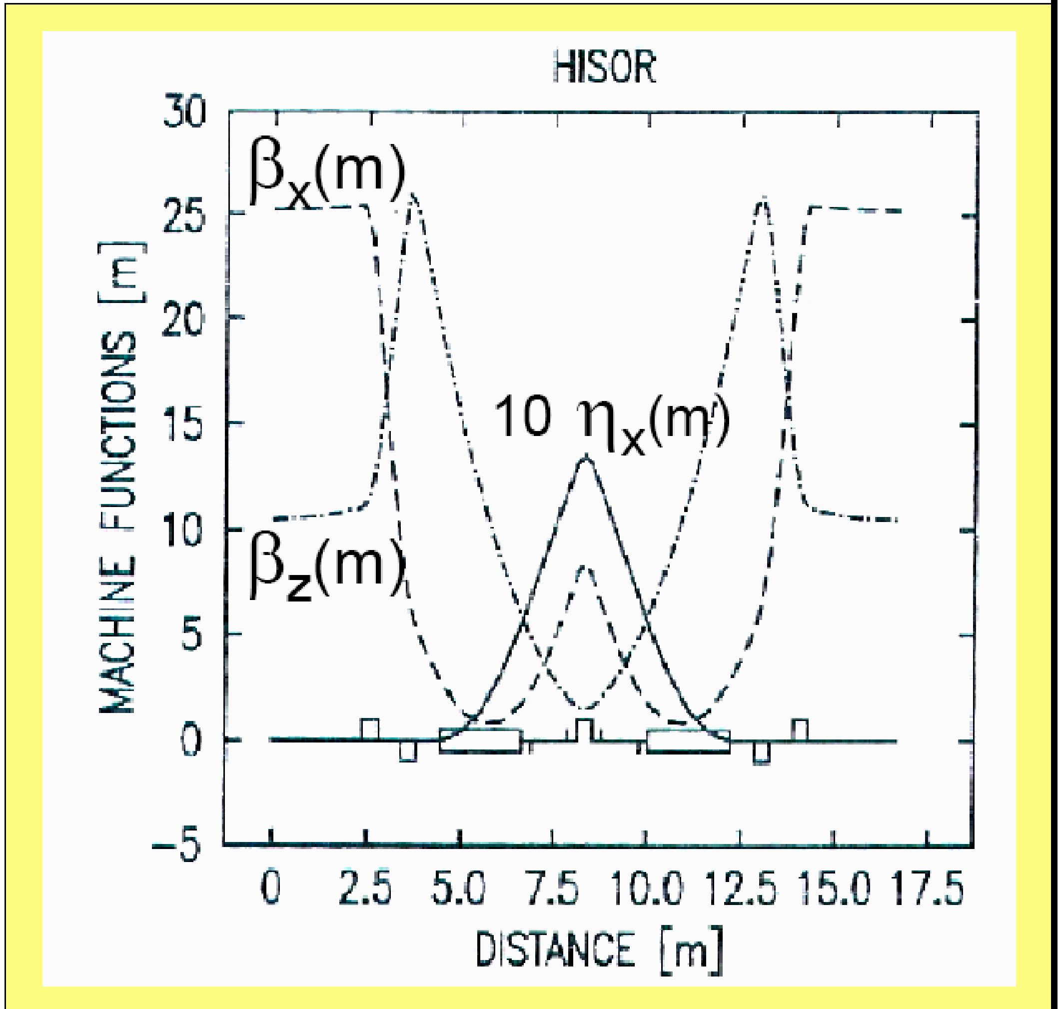
$$f = \frac{1}{2}(L_{\mathrm{drift}} + \frac{1}{2}L_{\mathrm{bend}})$$

  and the dispersion

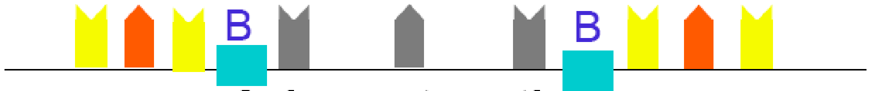
$$D_c = (L_{\mathrm{drift}} + \frac{1}{2}L_{\mathrm{bend}})\theta$$

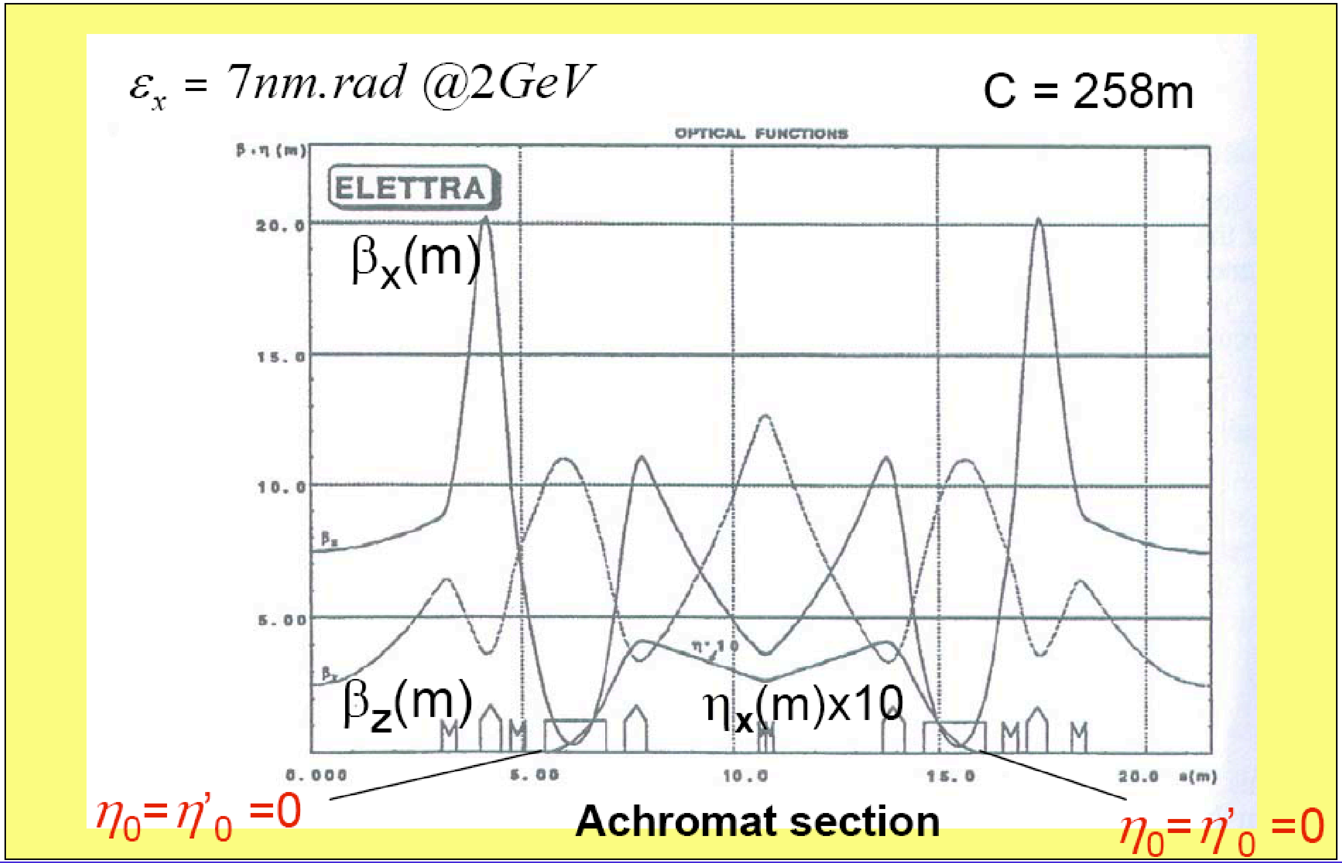
- Disadvantage the limited tunability and reduced space





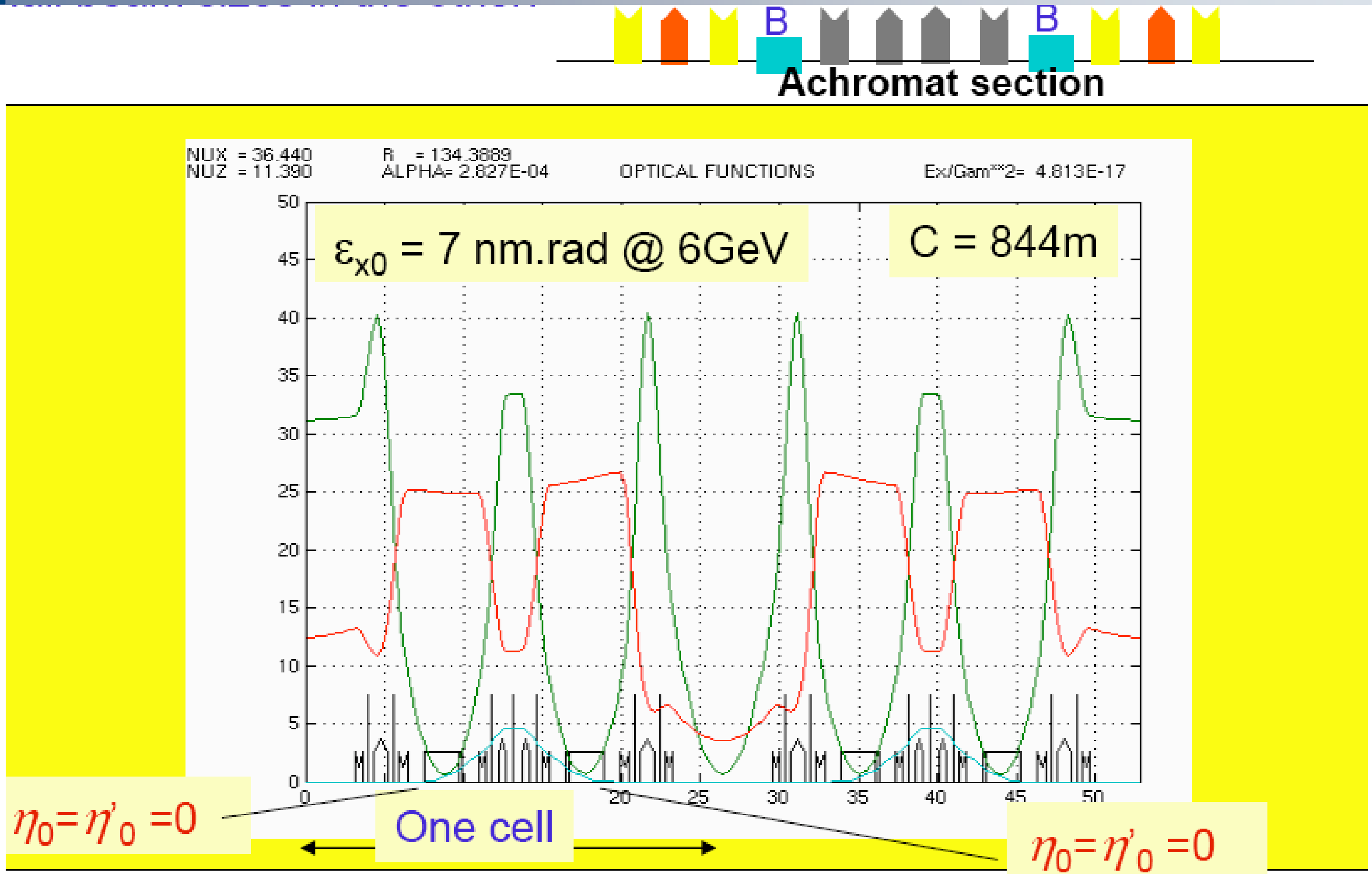
DBA with triplet





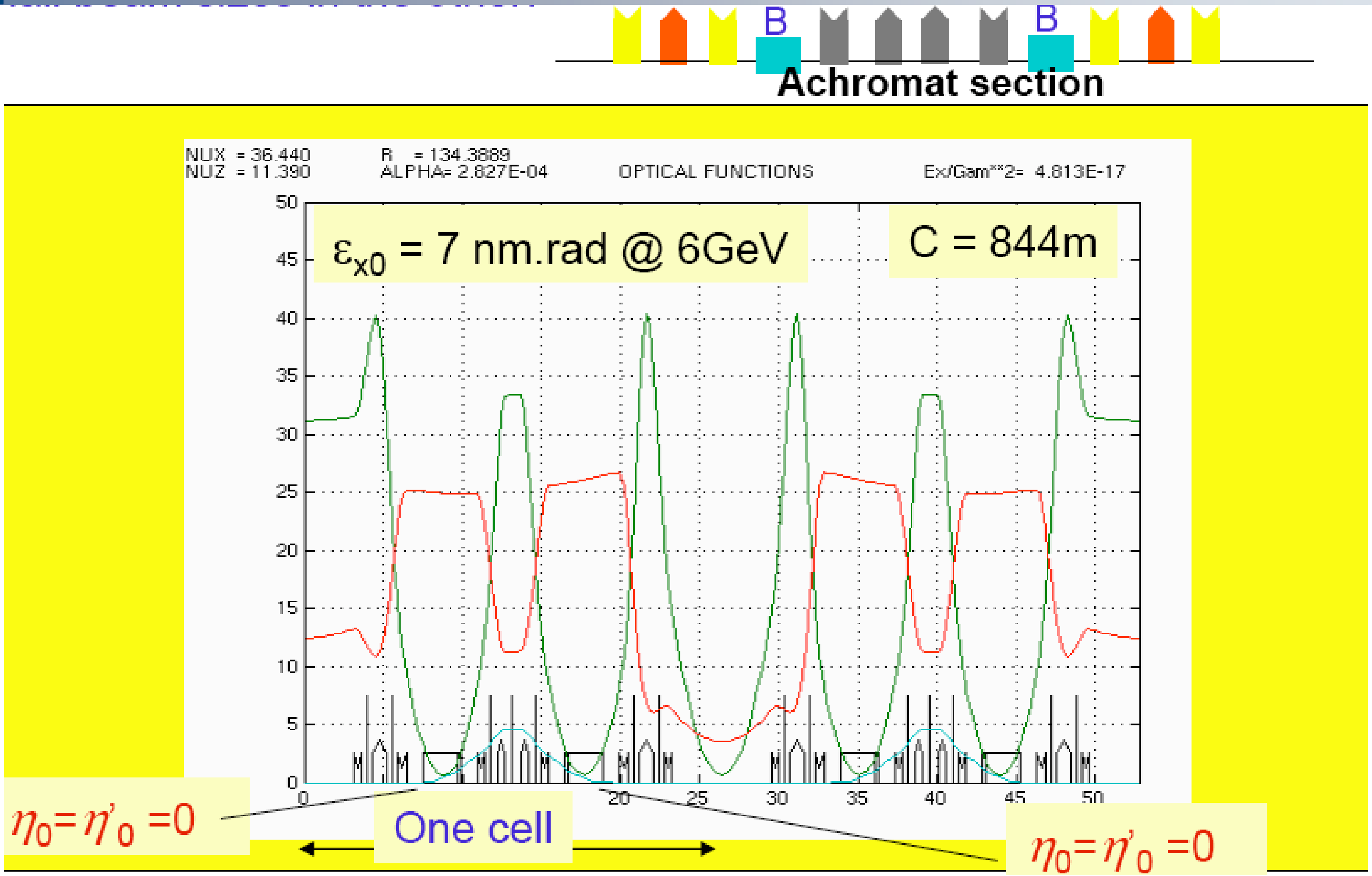
- Central triplet between the two bends and two triplets in the straight section to achieve the minimum emittance and achromatic condition
- Elettra (Trieste) uses this lattice achieving almost the absolute minimum emittance for an achromat
- Disadvantage the increased space in between the bends

Expanded DBA



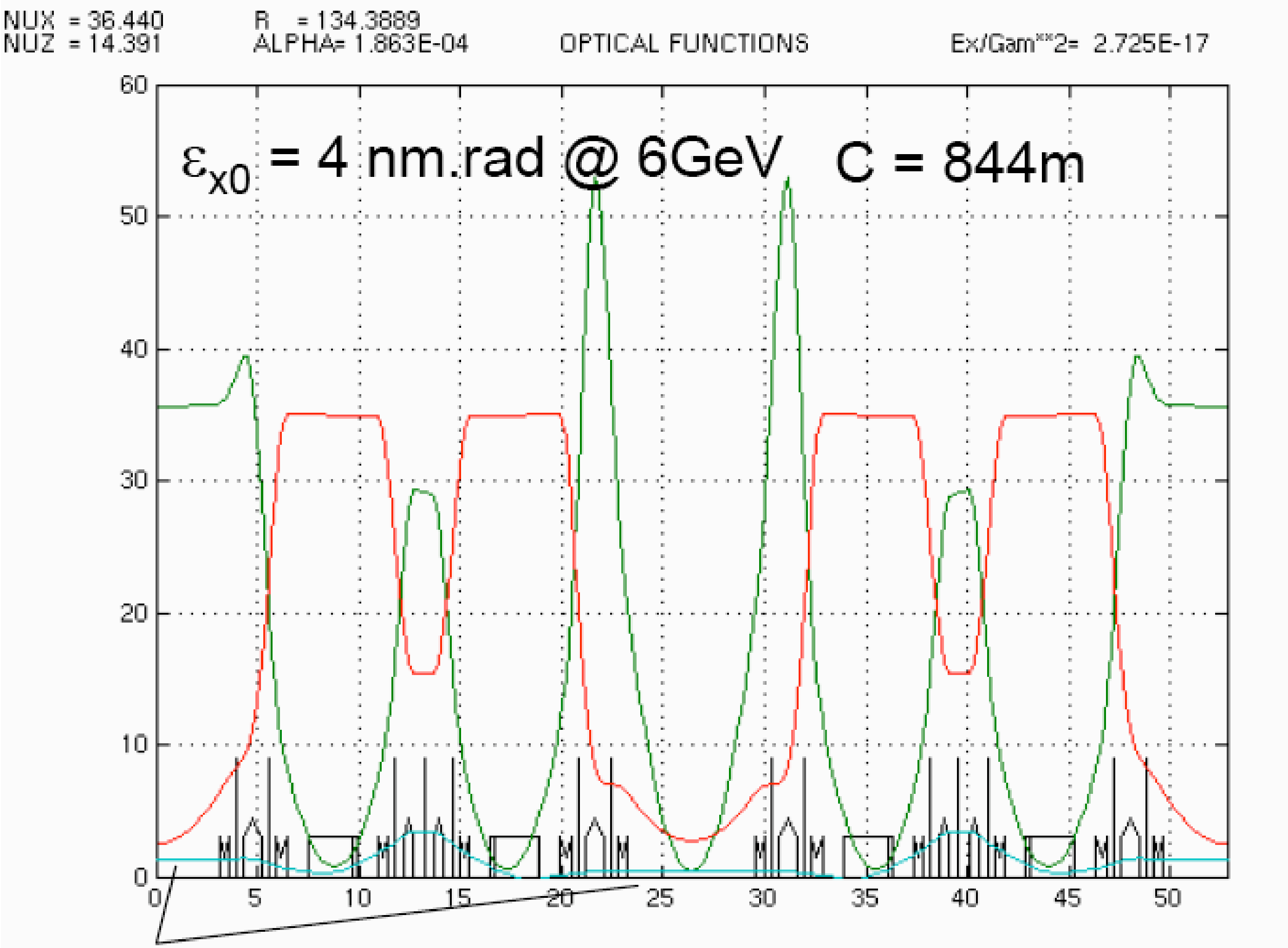
- Original lattice of ESRF storage ring, with 4 quadrupoles in between the bends
- Alternating moderate and low beta in intertions

Expanded DBA II



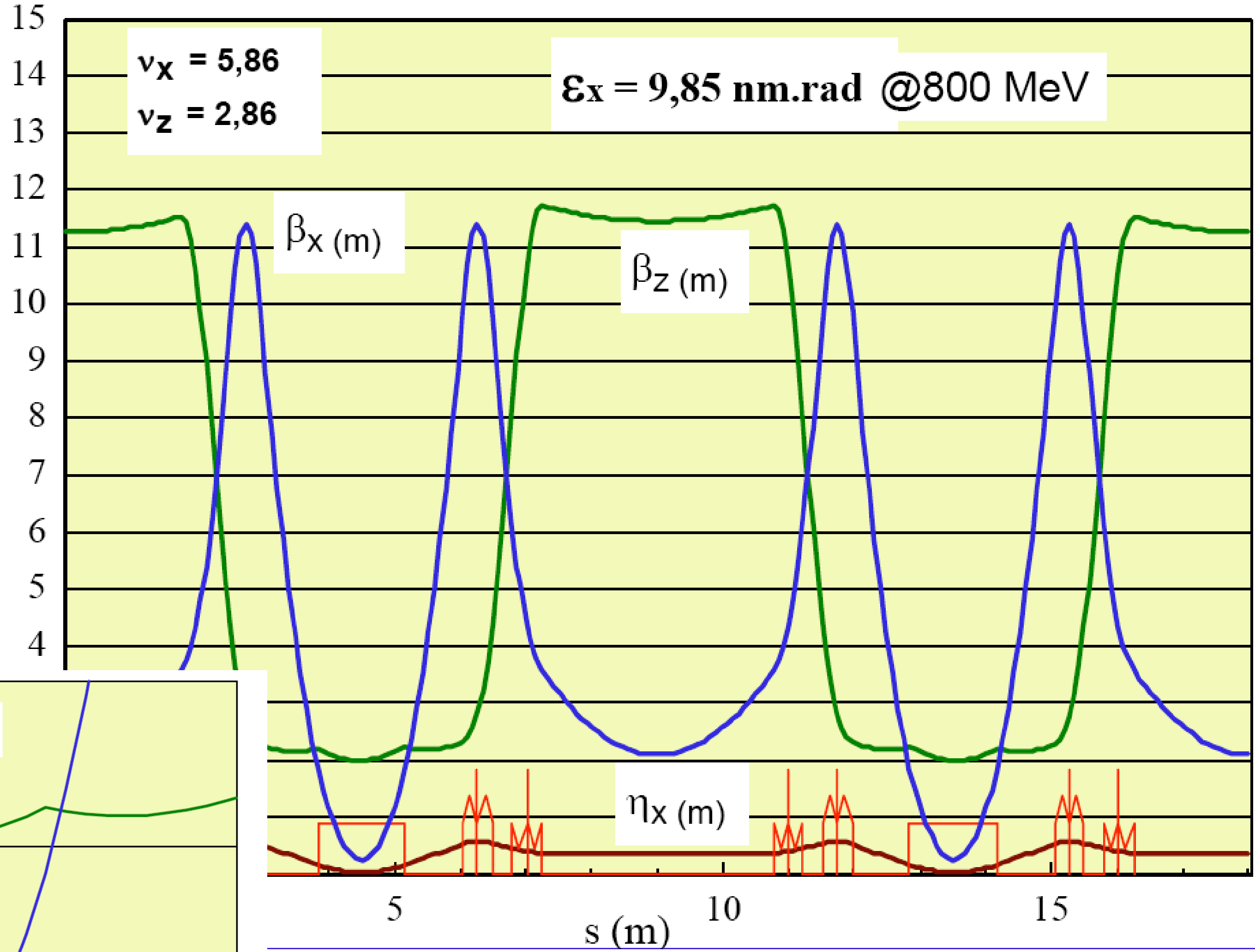
- Original lattice of ESRF storage ring, with 4 quadrupoles in between the bends
- Alternating moderate and low beta in insertions

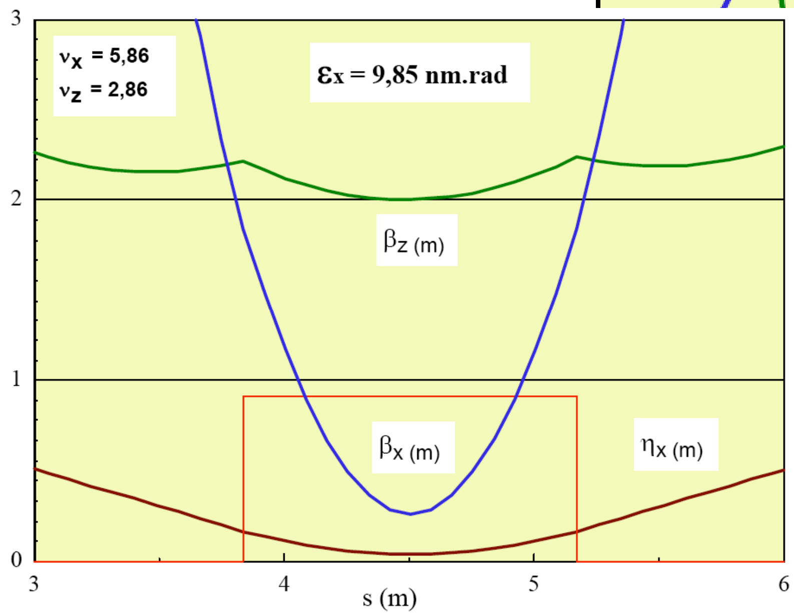
General double bend structure



- Reduce emittance by allowing dispersion in the straight sections
- ESRF reduced emittance almost halved the emittance achieved

Theoretical minimum emittance optics

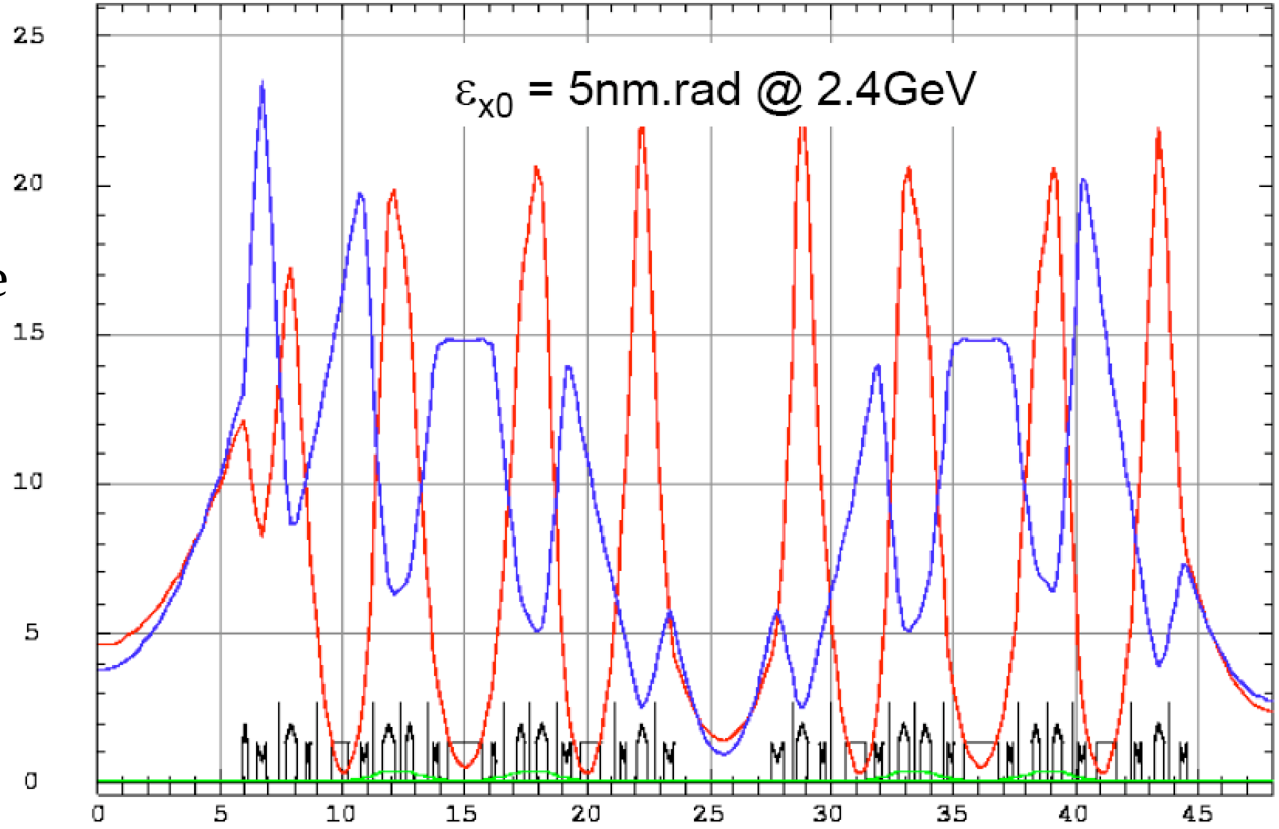




- Old Super-Aco ring could operate in a theoretical minimum emittance optics
- Structure mostly used in damping rings

Triple Bend Achromat

- Three bends with the central one with theoretical minimum emittance conditions
- Strict relationship between the bending angles and lengths of dipoles in order to achieve dispersion matching $\frac{L_2^3}{\rho_2^2} = 3\frac{L_1^3}{\rho_1^2}$
- A unique phase advance of 255° is needed for reaching the minimum emittance
- This minimum is equal to the one of the DBA
- Example, the Swiss Light Source



Light sources performance

Source	Energy (GeV)	Θ	C(m)	ΣL_{SS}	ε_{x0} (nm.rad)
ALS	1.9	0.1745	197	81	5.6
BESSYII	1.9	0.1963	240	89	6.4
DIAMOND	3	0.1309	562	218.2	2.74
ESRF	6	0.09817	844	201.6	4
ELETTRA	2	0.2618	258	74.78	7
SLS	2.4	0.2440	288	63	5
SOLEIL	2.75	0.1963	354	159.6	3.7

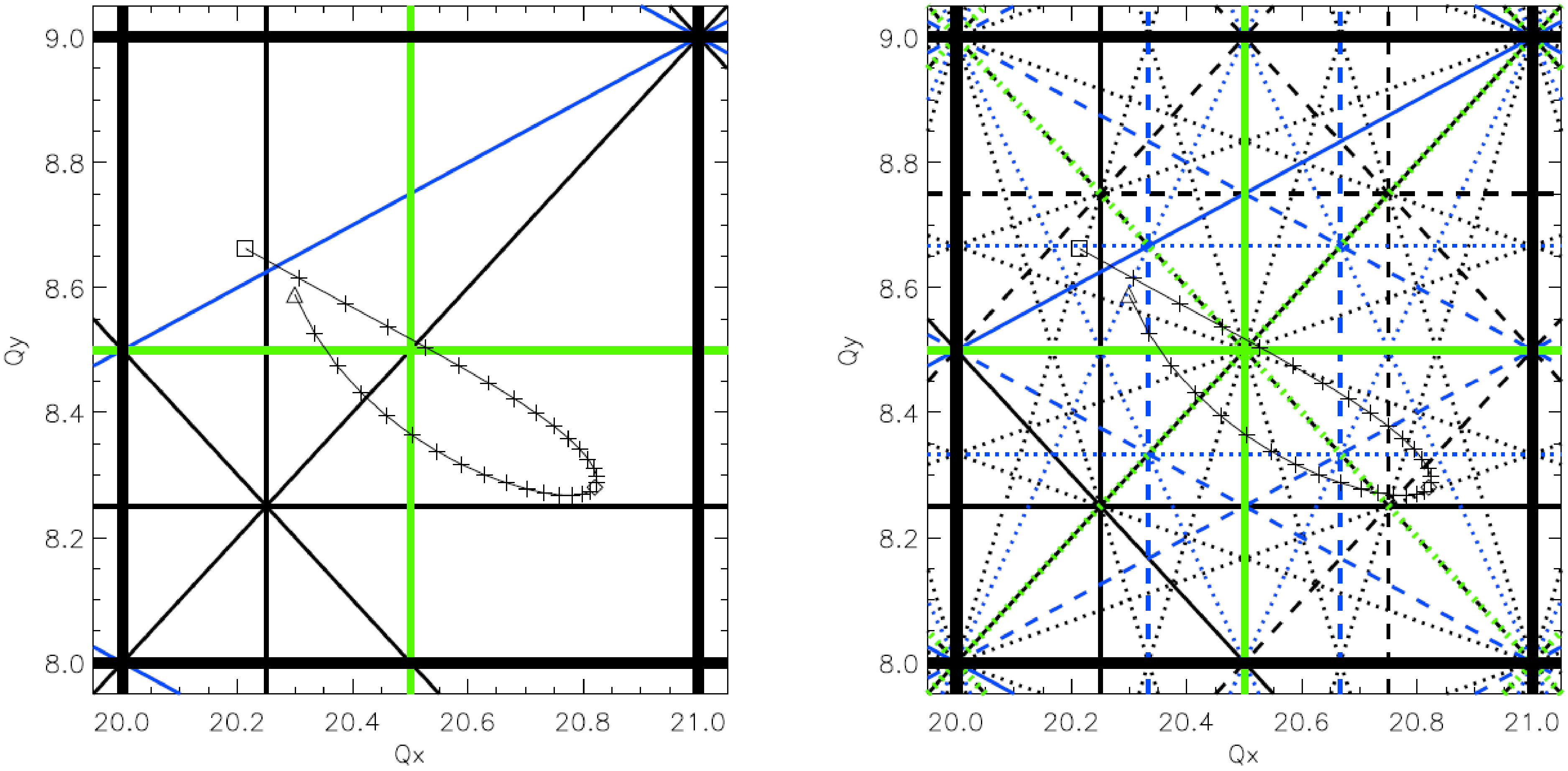
Circumference and periodicity

- Circumference choice implicates
  - Tunnel length should be small to reduce cost
  - Optics constraints necessitate circumference increase
  - Available spaces should not be reduce for all necessary equipment to fit
  - Sometimes it should be a multiple of the RF harmonic number and the RF wavelength
  - Varies from a few 1m to 27km (LEP)
- Large Periodicity implies
  - Simplicity in design and operation
  - Stability for dangerous resonance crossing (avoid only structural ones)
  - Reduction of cost for a few types of magnets
  - Varies from 1 (DORIS) to 40 (APS)

Tune and working point

- In a ring, the **tune** is defined from the 1-turn phase advance
$$Q_{x,y} = \frac{1}{2\pi} \oint \frac{ds}{\beta_{x,y}(s)}$$
i.e. number betatron oscillations per turn
- Taking the average of the betatron tune around the ring we have in **smooth approximation**
$$2\pi Q = \frac{C}{\langle\beta\rangle} \rightarrow Q = \frac{R}{\langle\beta\rangle}$$
- Extremely useful formula for deriving scaling laws
- The position of the tunes in a diagram of horizontal versus vertical tune is called a **working point**
- The tunes are imposed by the choice of the quadrupole strengths
- One should try to avoid **resonance conditions**

Ideal versus real lattice



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