Linear imperfections and correction

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Archamps, FRANCE

22-23 January 2013

References

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Outline

- Introduction: definitions and reminder
- Steering error and closed orbit distortion
- Focusing error and beta beating correction
- Linear coupling and correction
- Chromaticity

Equation reminder

Lorentz equation

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

E : Total Energy

T : Kinetic energy

p : Momentum

$$E = \sqrt{p^2 + m_0^2 c^4} = T + m_0 c^2 = T + E_0$$

** note that *p* is used instead of *cp*

β : reduced velocity

γ : reduced energy

$\beta\gamma$: reduced momentum

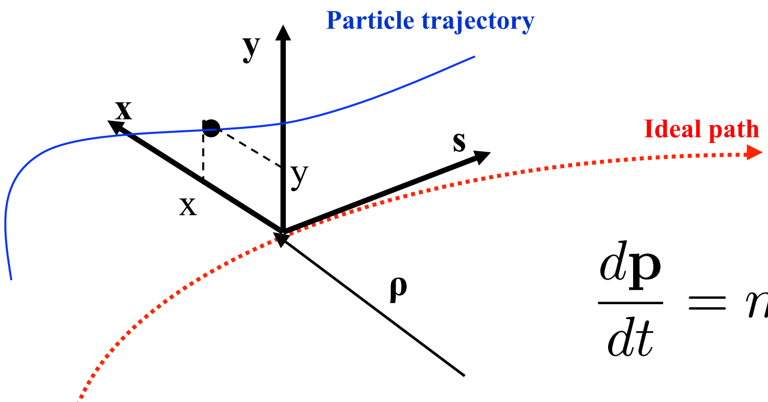
$$\beta = \frac{v}{c} \qquad \gamma = \frac{E}{m_0 c^2}$$

$$\beta\gamma = \frac{p}{m_0 c^2}$$

Reference trajectory

- Cartesian coordinates not useful to describe motion in a circular accelerator (not true for linacs)

- A system following an ideal path along the accelerator is used (**Frenet** reference system) $(\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_z}) \rightarrow (\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_s})$



- The curvature vector is $\boldsymbol{\kappa} = -\frac{d^2\mathbf{s}}{ds^2}$
- From Lorentz equation

$$\frac{d\mathbf{p}}{dt} = m_0\gamma\frac{d^2\mathbf{s}}{dt^2} = m_0\gamma v_s^2\frac{d^2\mathbf{s}}{ds^2} = -m_0\gamma v_s^2\boldsymbol{\kappa} = q[\mathbf{v}\times\mathbf{B}]$$

where we used the curvature vector definition and $\frac{d^2}{dt^2} = v_s^2\frac{d^2}{ds^2}$.

- By using $m_0\gamma v_s = p_s = (p^2 - p_x^2 - p_y^2)^{1/2} \approx p$, the ideal path of the **reference trajectory** is defined by

$$\boldsymbol{\kappa}_0 = -\frac{q}{p}\left[\frac{\mathbf{v}}{v_s}\times\mathbf{B_0}\right]$$

Beam guidance

- Consider uniform magnetic field $\mathbf{B} = \{0, B_y, 0\}$ in a direction perpendicular to particle motion. From the reference trajectory equation, after developing the cross product and considering that the transverse velocities v_x , $v_y \ll v_s$, the radius of curvature is

$$\frac{1}{\rho} = |k| = |\frac{q}{p}B| = |\frac{q}{\beta E}B|$$

- We define the **magnetic rigidity** $|B\rho| = \frac{p}{q}$

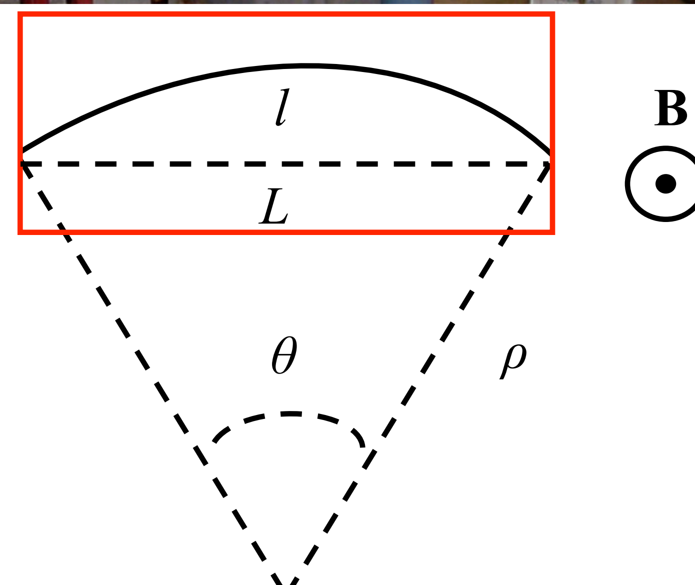
- In more practical units $\beta E[GeV] = 0.2998|B\rho|[Tm]$

- For ions with charge multiplicity *n* and atomic number *A*, the energy per nucleon is

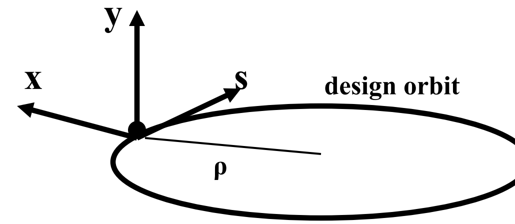
$$\beta \bar{E}[GeV/u] = 0.2998\frac{n}{A}|B\rho|[Tm]$$

Dipoles

- Consider ring for particles with energy E with N dipoles of length L (or effective length l, i.e. measured on beam path)
- **Bending angle** $$\theta = \frac{2\pi}{N}$$
- **Bending radius** $$\rho = \frac{l}{\theta}$$
- **Integrated dipole strength** $$Bl = \frac{2\pi}{N}\frac{\beta E}{q}$$
- Note:
  - By choosing a dipole field, the dipole length is imposed and vice versa
  - The higher the field, shorter or smaller number of dipoles can be used
  - Ring circumference (cost) is influenced by the field choice



Beam focusing

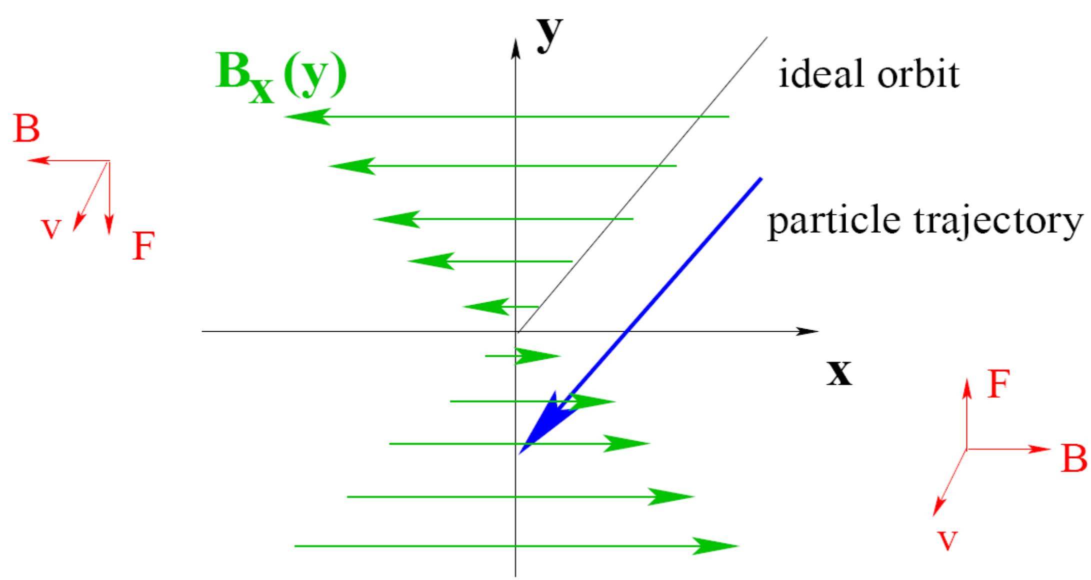


- Consider a particle in the design orbit.
- In the **horizontal plane**, it performs harmonic oscillations

$$x = x_0 \cos(\omega t + \phi) \quad \text{with frequency} \quad \omega = \frac{v_s}{\rho}$$

- The horizontal acceleration is described by $\frac{d^2x}{ds^2} = \frac{1}{v_s^2}\frac{d^2x}{dt^2} = -\frac{1}{\rho^2}x$
- There is a **week focusing** effect in the horizontal plane.
- In the **vertical plane**, the only force present is gravitation. Particles are displaced vertically following the usual law $\Delta y = \frac{1}{2}a_g \Delta t^2$
- Setting a_g = 10 m/s^2, the particle is displaced by **18mm** (LHC dipole aperture) in **60ms** (a few hundreds of turns in LHC)

Need of **focusing**!



Quadrupoles

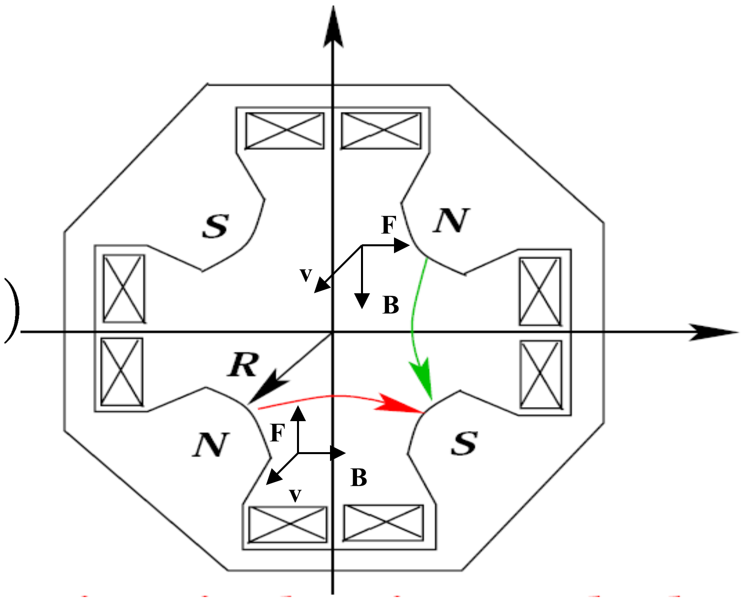
- Quadrupoles are focusing in one plane and defocusing in the other
- The field is $(B_x, B_y) = G(y, x)$
- The resulting force $(F_x, F_y) = k(y, -x)$ with the normalised gradient defined as

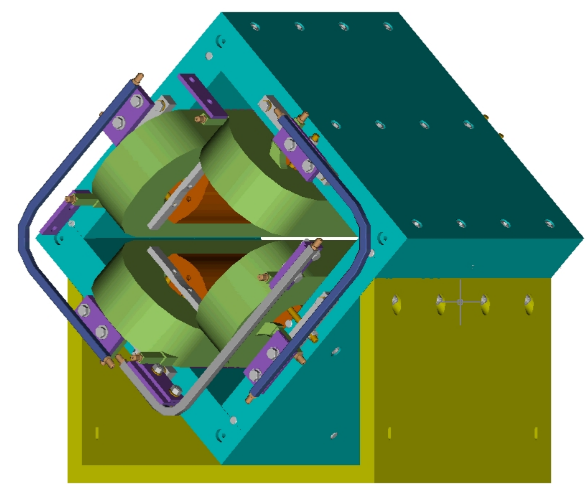
$$k = \frac{qG}{\beta E}$$

- In more practical units,

$$k[m^{-2}] = 0.2998 \frac{G[T/m]}{\beta E[GeV]}$$

- Need to alternate focusing and defocusing in order to control the beam, i.e. **alternating gradient focusing**





Equations of motion – Linear fields

- Consider *s*-dependent fields from dipoles and normal quadrupoles

$$B_y = B_0(s) - G(s)x \ , \quad B_x = -G(s)y$$

- The total momentum can be written $p = p_0(1 + \frac{\Delta p}{p})$
- With magnetic rigidity $B_0\rho = \frac{p_0}{q}$ and normalized gradient

$$k(s) = \frac{G(s)}{B_0\rho}$$

the equations of motion are

$$x'' - \left(k(s) + \frac{1}{\rho(s)^2}\right) x = \frac{1}{\rho(s)} \frac{\Delta p}{p}$$

$$y'' + k(s)\ y = 0$$

- Inhomogeneous equations with *s*-dependent coefficients
- The term $\frac{1}{\rho^2}$ corresponds to the dipole **week focusing** and

$\frac{1}{\rho} \frac{\Delta p}{p}$ respresents **off-momentum** particles

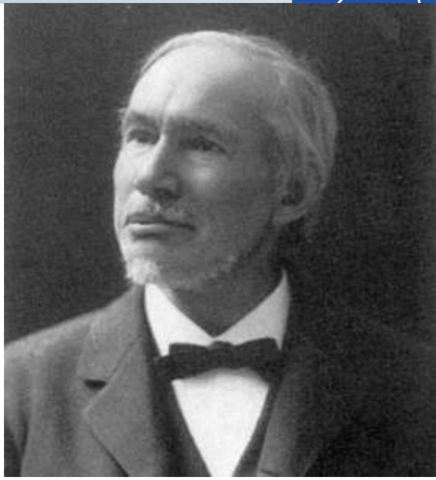
Hill's equations

- Solutions are combination of the homogeneous and inhomogeneous equations' solutions

- Consider particles with the design momentum. The equations of motion become

$$
\begin{aligned}
x'' + K_x(s)\ x &= 0 \\
y'' + K_y(s)\ y &= 0
\end{aligned}
$$

with $K_x(s) = -\left(k(s) - \frac{1}{\rho(s)^2}\right)$, $K_y(s) = k(s)$



George Hill

- **Hill's equations of linear transverse particle motion**

- Linear equations with *s*-dependent coefficients (harmonic oscillator with time dependent frequency)

- In a ring (or in transport line with symmetries), coefficients are periodic $K_x(s) = K_x(s+C)$, $K_y(s) = K_y(s+C)$

- Not straightforward to derive analytical solutions for whole accelerator

Betatron motion

- The on-momentum linear betatron motion of a particle in both planes, is described by

$$u(s) = \sqrt{\epsilon\beta(s)}\cos(\psi(s)+\psi_0) \quad u \mapsto \{x, y\}$$

with $\alpha,\ \beta,\ \gamma$ the twiss functions $\quad \alpha(s) = -\frac{\beta(s)'}{2},\quad \gamma = \frac{1+\alpha(s)^2}{\beta(s)}$

ψ the **betatron phase** $\quad \psi(s) = \int \frac{ds}{\beta(s)}$

and the **beta function** β is defined by the **envelope equation**

$$2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4$$

- By differentiation, we have that the **angle** is

$$u'(s) = -\sqrt{\frac{\epsilon}{\beta(s)}}\left(\sin(\psi(s)+\psi_0) + \alpha(s)\cos(\psi(s)+\psi_0)\right)$$

General transfer matrix

- From the position and angle equations,

$$\cos(\psi(s)+\psi_0) = \frac{u}{\sqrt{\epsilon\beta(s)}}\ , \quad \sin(\psi(s)+\psi_0) = \sqrt{\frac{\beta(s)}{\epsilon}}u' + \frac{\alpha(s)}{\sqrt{\epsilon\beta(s)}}u$$

- Expand the trigonometric formulas and set $\psi(0)=0$ to get the transfer matrix from location 0 to s

$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \mathcal{M}_{0\rightarrow s}\begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

with

$$\mathcal{M}_{0\rightarrow s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\psi + \alpha_0\sin\Delta\psi) & \sqrt{\beta(s)\beta_0}\sin\Delta\psi \\ \frac{(a_0 - a(s))\cos\Delta\psi - (1+\alpha_0\alpha(s))\sin\Delta\psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}(\cos\Delta\psi - \alpha_0\sin\Delta\psi) \end{pmatrix}$$

and $\mu(s) = \Delta\psi = \int_0^s \frac{ds}{\beta(s)}$ the **phase advance**

Periodic transfer matrix

- Consider a periodic cell of length C
- The optics functions are $\beta_0 = \beta(C) = \beta\ , \quad \alpha_0 = \alpha(C) = \alpha$

  and the phase advance $$\mu = \int_0^C \frac{ds}{\beta(s)}$$

- The transfer matrix is

$$\mathcal{M}_C = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- The cell matrix can be also written as

$$\mathcal{M}_C = \mathcal{I}\cos\mu + \mathcal{J}\sin\mu$$

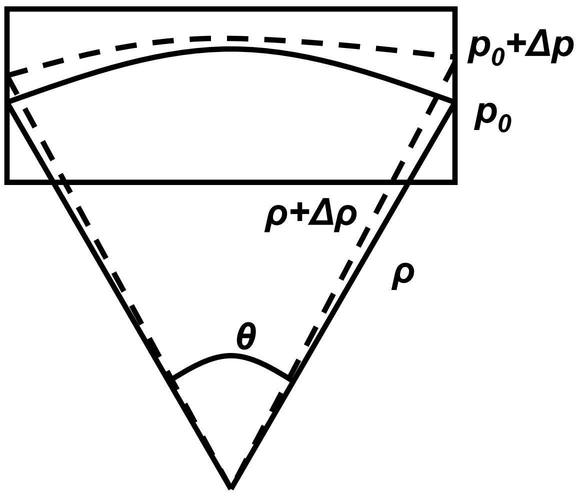
with $\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and the **Twiss matrix** $\mathcal{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$

Tune and working point

- In a ring, the **tune** is defined from the 1-turn phase advance
$$Q_{x,y} = \frac{1}{2\pi}\oint \frac{ds}{\beta_{x,y}(s)} = \frac{\nu_{x,y}}{2\pi}$$
i.e. number betatron oscillations per turn
- Taking the average of the betatron tune around the ring we have in **smooth approximation**
$$\nu = 2\pi Q = \frac{C}{\langle\beta\rangle} \rightarrow Q = \frac{R}{\langle\beta\rangle}$$
- Extremely useful formula for deriving scaling laws
- The position of the tunes in a diagram of horizontal versus vertical tune is called a **working point**
- The tunes are imposed by the choice of the quadrupole strengths
- One should try to avoid **resonance conditions**

Effect of dipole on off-momentum particles

- Up to now all particles had the same momentum p_0
- What happens for off-momentum particles, i.e. particles with momentum $p_0+\Delta p$?
- Consider a dipole with field B and bending radius ρ
- Recall that the magnetic rigidity is $B\rho = \frac{p_0}{q}$ and for off-momentum particles



$$B(\rho+\Delta\rho) = \frac{p_0+\Delta p}{q} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p_0}$$

- Considering the effective length of the dipole unchanged

$$\theta\rho = l = \text{const.} \Rightarrow \rho\Delta\theta + \theta\Delta\rho = 0 \Rightarrow \frac{\Delta\theta}{\theta} = -\frac{\Delta\rho}{\rho} = -\frac{\Delta p}{p_0}$$

- Off-momentum particles get different deflection (different orbit)

$$\Delta\theta = -\theta\frac{\Delta p}{p_0}$$

Dispersion equation

- Consider the equations of motion for off-momentum particles

$$x'' + K_x(s)x = \frac{1}{\rho(s)}\frac{\Delta p}{p}$$

- The solution is a sum of the **homogeneous** (on-momentum) and the **inhomogeneous** (off-momentum) equation solutions

$$x(s) = x_H(s) + x_I(s)$$

- In that way, the equations of motion are split in two parts

$$\begin{aligned} x''_H + K_x(s)x_H &= 0 \\ x''_I + K_x(s)x_I &= \frac{1}{\rho(s)}\frac{\Delta p}{p} \end{aligned}$$

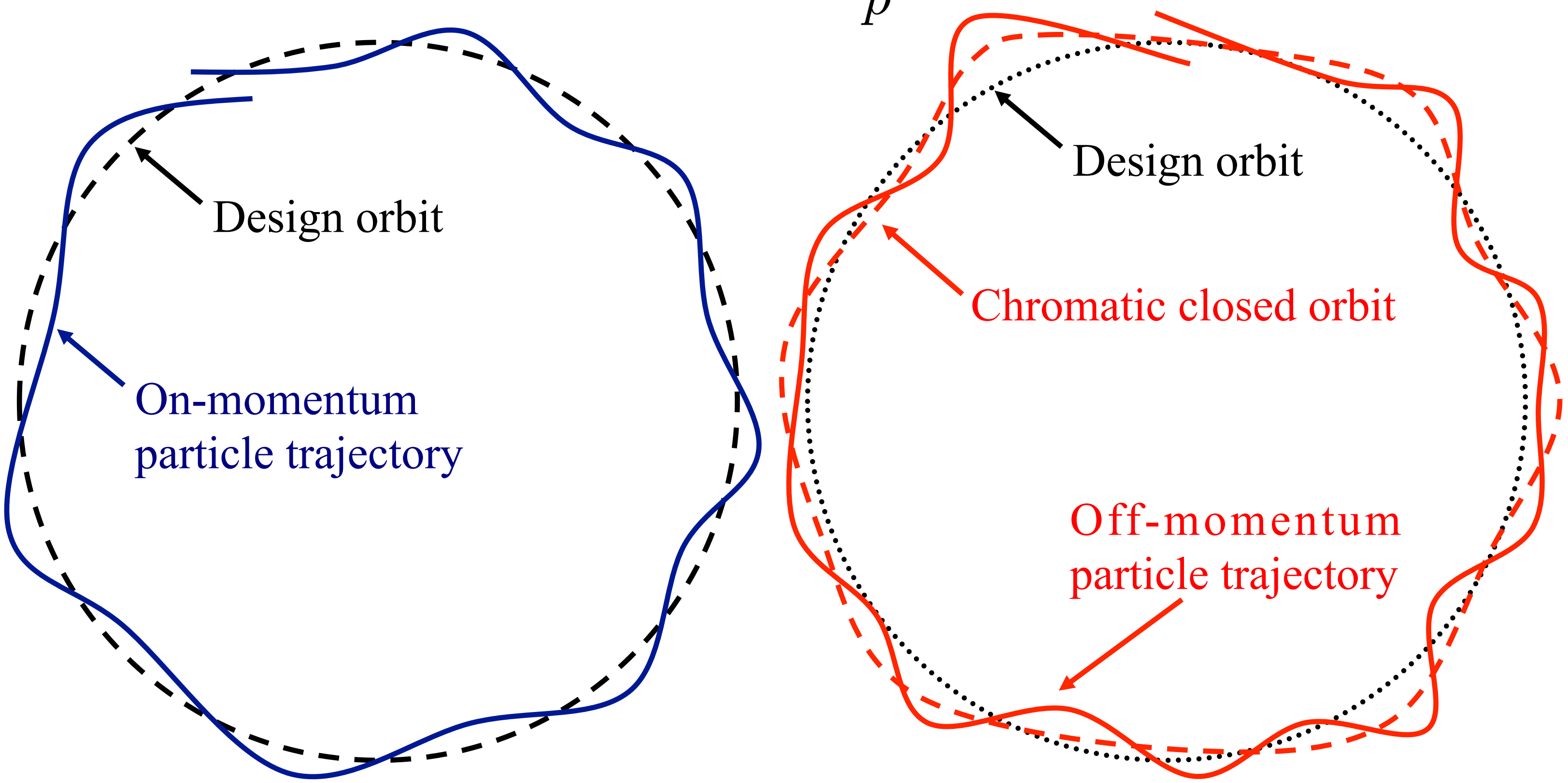
- The **dispersion function** can be defined as $D(s) = \frac{x_I(s)}{\Delta p/p}$
- The dispersion equation is

$$D''(s) + K_x(s)\ D(s) = \frac{1}{\rho(s)}$$

Closed orbit

- Design orbit defined by main dipole field
- On-momentum particles oscillate around design orbit
- Off-momentum particles are not oscillating around design orbit, but around "chromatic" closed orbit
- Distance from the design orbit depends linearly to momentum spread and dispersion

$$x_D = D(s)\frac{\Delta p}{p}$$



Beam orbit stability

- Beam orbit stability very critical
  - Injection and extraction efficiency of synchrotrons
  - Stability of collision point in colliders
  - Stability of the synchrotron light spot in the beam lines of light sources
- Consequences of orbit distortion
  - Miss-steering of beams, modification of the dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling of beam motion, modulation of lattice functions, poor injection and extraction efficiency
- Causes
  - Long term (Years - months)
    - Ground settling, season changes
  - Medium (Days –Hours)
    - Sun and moon, day-night variations (thermal), rivers, rain, wind, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns
  - Short (Minutes - Seconds)
    - Ground vibrations, power supplies, injectors, experimental magnets, air conditioning, refrigerators/compressors, water cooling

Closed orbit distortion

- Magnetic imperfections distorting the orbit
  - Dipole field errors (or energy errors)
  - Dipole rolls
  - Quadrupole misalignments
    - Consider the displacement of a particle δx from the ideal orbit . The vertical field in the quadrupole is

$$B_y = G\bar{x} = G(x + \delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

- Remark: Dispersion creates a closed orbit distortion for off-momentum particles with $\delta x = D(s)\dfrac{\delta p}{p}$
- Effect of orbit errors in any multi-pole magnet

$$B_y = b_n \bar{x}^n = b_n (x + \delta x)^n = b_n (\underbrace{x^n}_{\textbf{2(n+1)-pole}} + \underbrace{n \delta x x^{n-1}}_{\textbf{2n-pole}} + \underbrace{\frac{n(n-1)}{2}(\delta x)^2 x^{n-2}}_{\textbf{2(n-1)-pole}} + \cdots + \underbrace{(\delta x)^n}_{\textbf{dipole}})$$

- **Feed-down**

Effect of single dipole kick

- Consider a single dipole kick $\theta = \delta u_0' = \delta u'(s_0) = \frac{\delta(Bl)}{B\rho}$ at $s=s_0$
- The coordinates before and after the kick are

$$\begin{pmatrix} u_0 \\ u_0' - \theta \end{pmatrix} = \mathcal{M} \begin{pmatrix} u_0 \\ u_0' \end{pmatrix}$$

with the 1-turn transfer matrix

$$\mathcal{M} = \begin{pmatrix} \cos 2\pi Q + \alpha_0 \sin 2\pi Q & \beta_0 \sin 2\pi Q \\ -\gamma_0 \sin 2\pi Q & \cos 2\pi Q - \alpha_0 \sin 2\pi Q \end{pmatrix}$$

- The final coordinates are $u_0 = \theta \frac{\beta_0}{2 \tan \pi Q}$ and $u_0' = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan \pi Q}\right)$
- For any location around the ring it can be shown that

$$u(s) = \theta \underbrace{\frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)}} \cos(\pi Q - |\psi(s) - \psi_0|)$$

Maximum distortion amplitude

Transport of orbit distortion due to dipole kick

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$\begin{aligned} u_2 &= m_{11}u_1 + m_{12}u_1' \\ u_2' &= m_{21}u_1 + m_{22}u_1' \end{aligned}$$

- Consider a single dipole kick at position 1 $\theta_1 = \dfrac{\delta(Bl)}{B\rho}$
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u_1' + \theta_1) \rightarrow \delta u_2 = m_{12}\theta_1$$

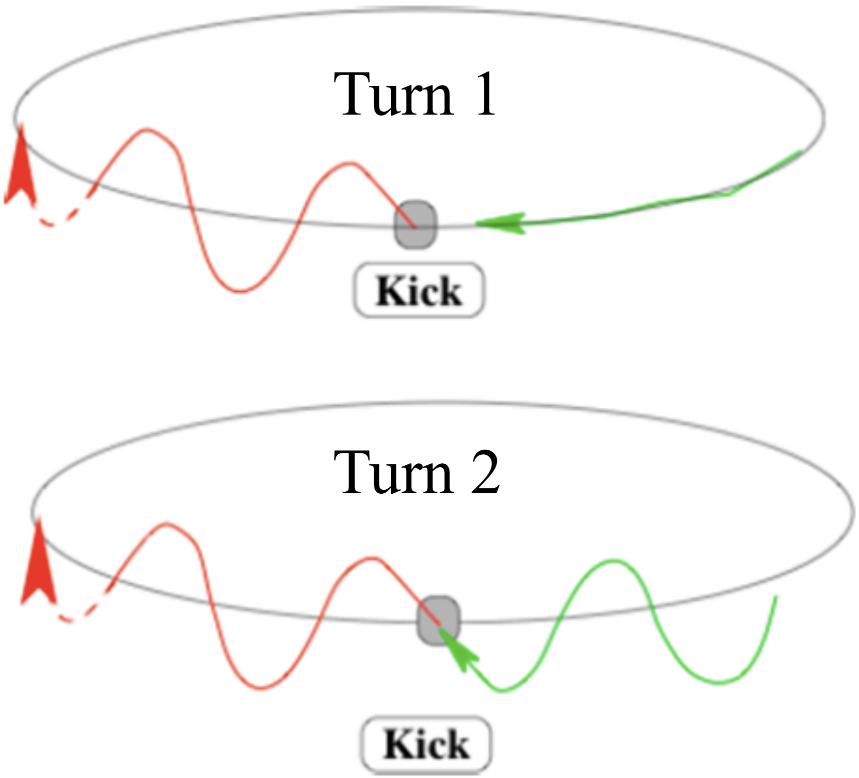
- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\theta_1$$

$$\delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}}\left[\cos(\psi_{12})\theta_1 - \alpha_2\sin(\psi_{12})\right]$$

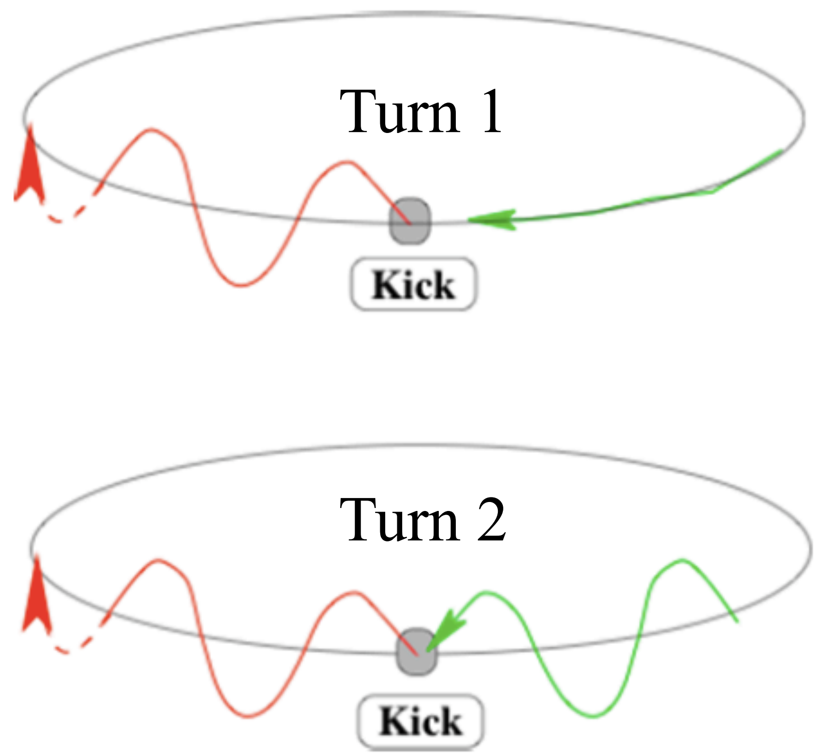
Integer and half integer resonance

- Dipole perturbations add-up in consecutive turns for $Q = n$
- Integer tune excites orbit oscillations (resonance)



$$\delta u_2 = \sqrt{\beta_1 \beta_2} \sin(\psi_{12})\theta_1$$

- Dipole kicks get cancelled in consecutive turns for $Q = n/2$
- Half-integer tune cancels orbit oscillations



$$\delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}} \left[\cos(\psi_{12})\theta_1 - \alpha_2 \sin(\psi_{12})\right]$$

Global orbit distortion

- Orbit distortion due to many errors

Courant and Snyder, 1957

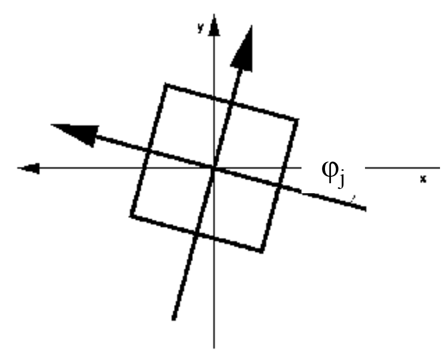
$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+C} \theta(\tau)\sqrt{\beta(\tau)} \cos(\pi Q - |\psi(s) - \psi(\tau)|) d\tau$$

- By approximating the errors as delta functions in *n* locations, the distortion at *i* observation points (Beam Position Monitors) is

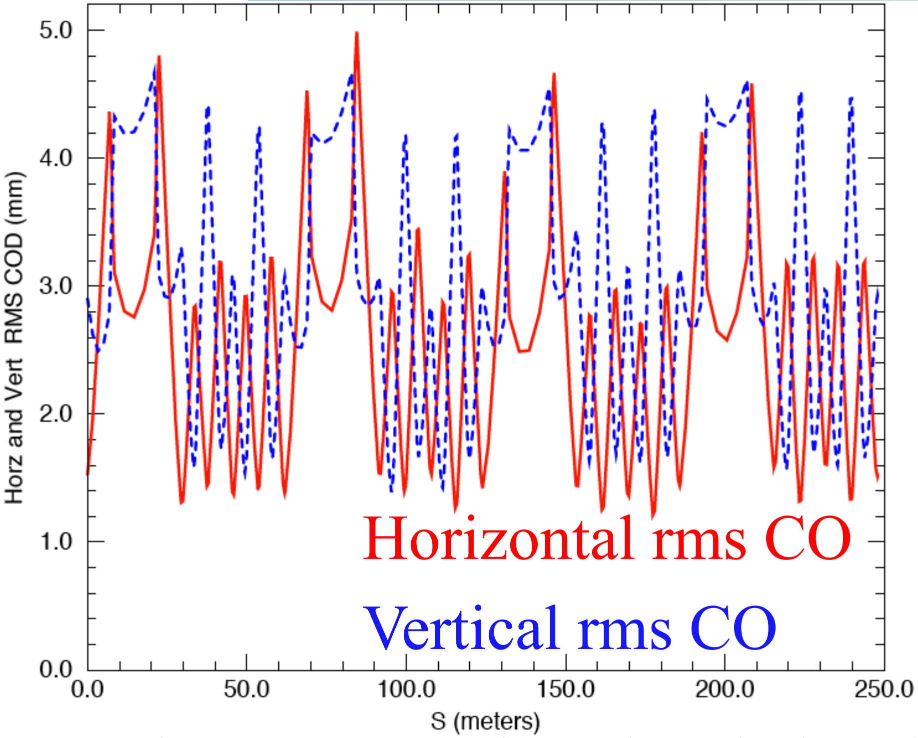
$$u_i = \frac{\sqrt{\beta_i}}{2\sin(\pi Q)} \sum_{j=i+1}^{i+n} \theta_j \sqrt{\beta_j} \cos(\pi Q - |\psi_i - \psi_j|)$$

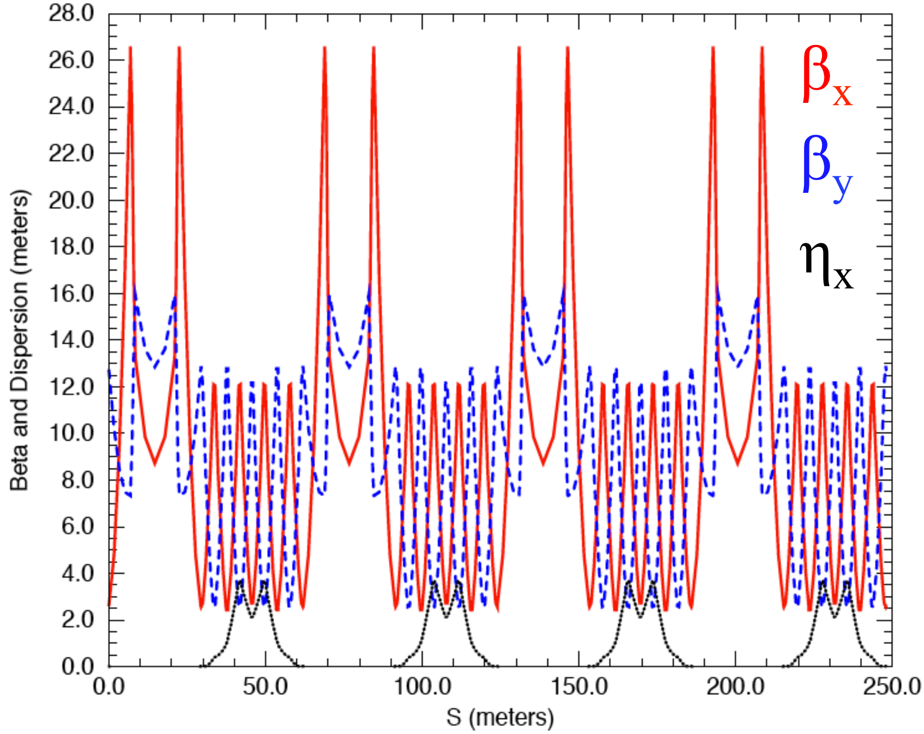
with the kick produced by the *j*th error

  - Integrated dipole field error $\theta_j = \frac{\delta(B_j l_j)}{B\rho}$
  - Dipole roll $\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$
  - Quadrupole displacement $\theta_j = \frac{G_j l_j \delta u_j}{B\rho}$



Example: Orbit distortion for the SNS ring





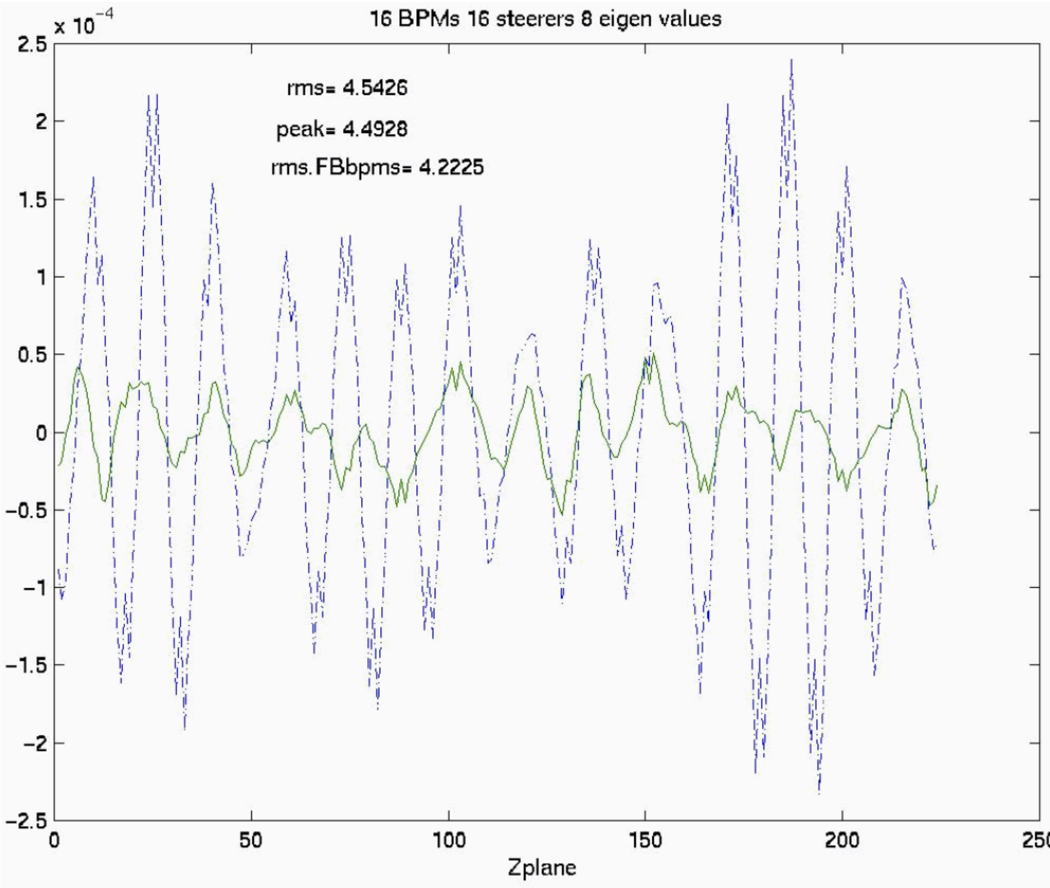
- In the SNS accumulator ring, the beta function is **6m** in the dipoles and **30m** in the quadrupoles.
- Consider dipole error of **1mrad**
- The tune is **6.2**
- The maximum orbit distortion in the dipoles is $u_0 = \frac{\sqrt{6 \cdot 6}}{2\sin(6.2\pi)} \cdot 10^{-3} \approx 5\text{mm}$
- For quadrupole displacement giving the same **1mrad** kick (and betas of 30m) the maximum orbit distortion is 25mm, to be compared to magnet radius of 105mm

Example: Orbit distortion in ESRF storage ring

- In the ESRF storage ring, the beta function is **1.5m** in the dipoles and **30m** in the quadrupoles.
- Consider dipole error of **1mrad**
- The horizontal tune is **36.44**
- Maximum orbit distortion in dipoles

$$u_0 = \frac{\sqrt{1.5 \cdot 1.5}}{2\sin(36.44\pi)} \cdot 10^{-3} \approx 1\text{mm}$$

- For quadrupole displacement with **1mm**, the distortion is $u_0 \approx 8\text{mm}$!!!
- Magnet alignment is critical



Vertical orbit correction with 16BPMs and steerers

Statistical estimation of orbit errors

- Consider random distribution of errors in N magnets
- The expectation (rms) value is given by

$$u_{\rm rms}(s) = \frac{\sqrt{\beta(s)}}{2\sqrt{2}|\sin(\pi Q)|}\left(\sum_i \sqrt{\beta_i}\theta_i\right)_{\rm rms} = \frac{\sqrt{N\beta(s)\beta_{\rm rms}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\rm rms}$$

- Example:
  - In the SNS ring, there are **32** dipoles and **54** quadrupoles
  - The rms value of the orbit distortion in the dipoles

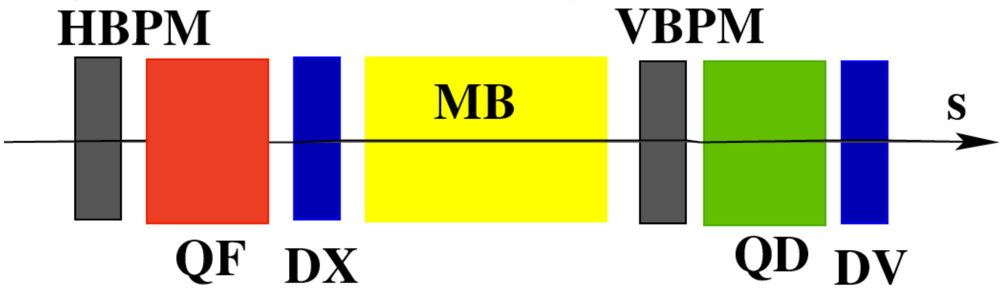
$$u_{\rm rms}^{\rm dip} = \frac{\sqrt{6\cdot 6}\sqrt{32}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 2{\rm cm}$$

  - In the quadrupoles, for equivalent kick

$$u_{\rm rms}^{\rm quad} = \frac{\sqrt{30\cdot 30}\sqrt{54}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 13{\rm cm}$$

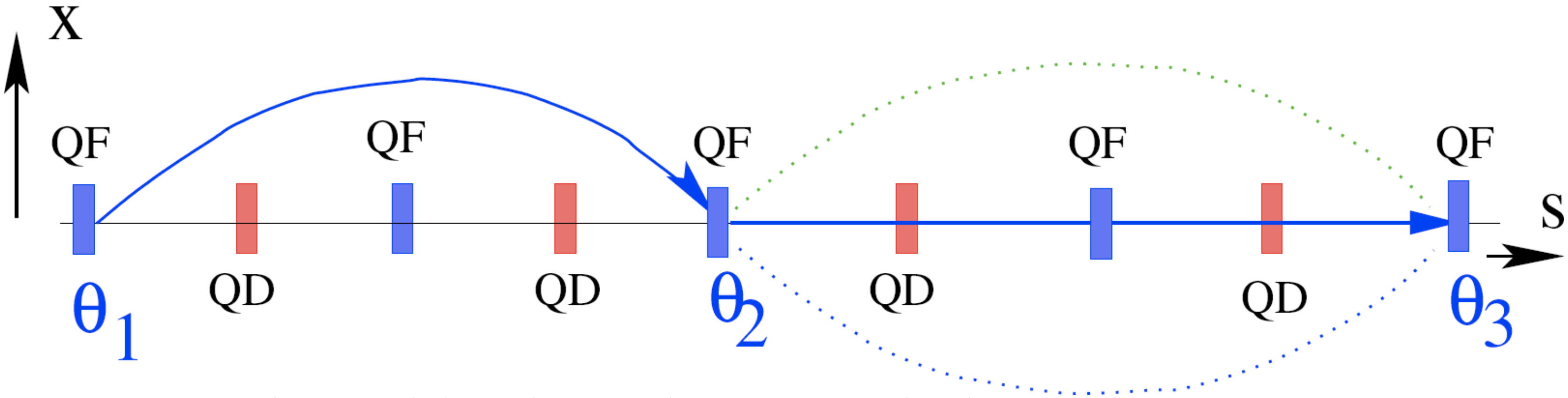
Correcting the orbit distortion

- Place horizontal and vertical dipole correctors close to focusing and defocusing quads, respectively



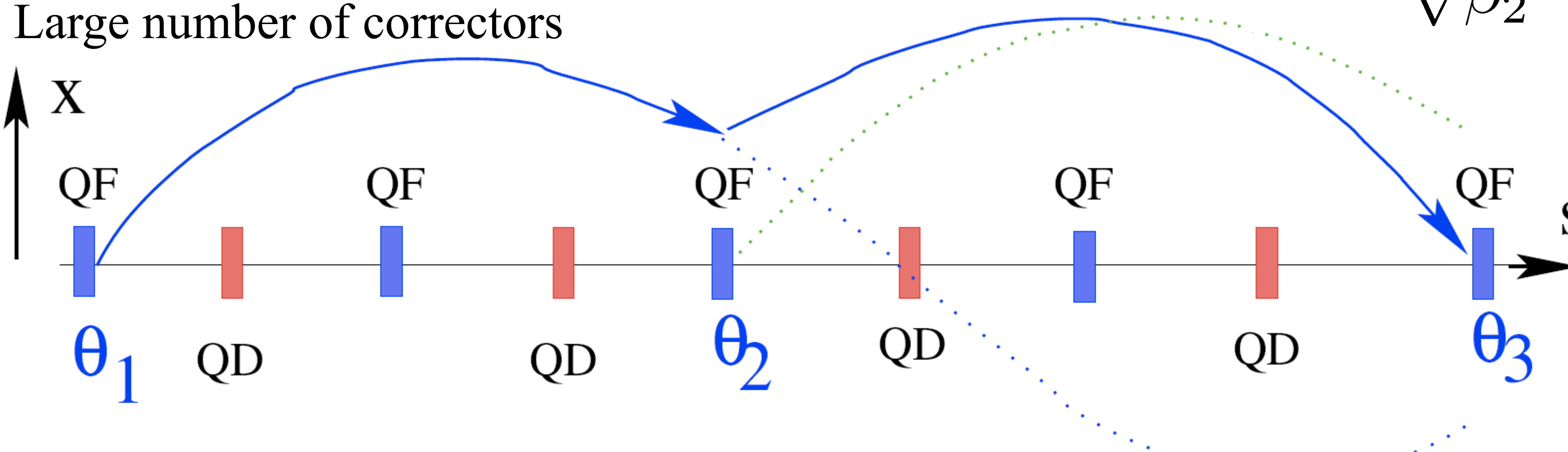
- Simulate (random distribution of errors) or measure orbit in BPMs
- Minimize orbit distortion
  - Globally
    - Harmonic , minimizing components of the orbit frequency response after a Fourier analysis
    - Most efficient corrector (MICADO), finding the most efficient corrector for minimizing the rms orbit
    - Least square minimization using the orbit response matrix of the correctors
  - Locally
    - Sliding Bumps
    - Singular Value Decomposition (SVD)

Orbit bumps



- **2-bump**: Only good for phase advance equal π between correctors
- Sensitive to lattice and BPM errors
- Large number of correctors

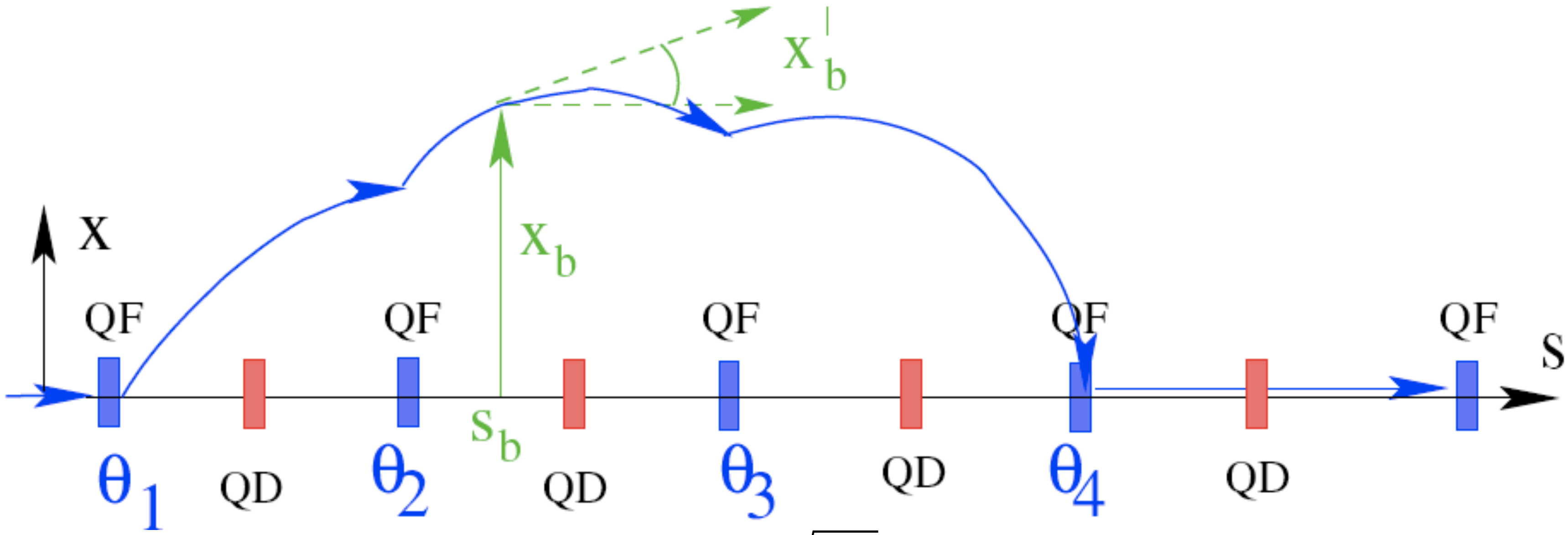
$$\theta_2 = \frac{\sqrt{\beta_1}}{\sqrt{\beta_2}}\theta_1$$



- **3-bump**: works for any lattice
- Need large number of correctors
- No control of angles (need 4 bumps)

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

4-bump



$$\theta_1 = \frac{1}{\sqrt{\beta_1\beta_s}}\frac{\cos\psi_{2s} - \alpha_s \sin\psi_{2s}}{\sin\psi_{12}}x_b - \sqrt{\frac{\beta_s}{\beta_1}\frac{\sin\psi_{2s}}{\sin\psi_{12}}}x'_b$$

$$\theta_2 = \frac{1}{\sqrt{\beta_2\beta_s}}\frac{\cos\psi_{1s} - \alpha_s \sin\psi_{1s}}{\sin\psi_{12}}x_b + \sqrt{\frac{\beta_s}{\beta_2}\frac{\sin\psi_{1s}}{\sin\psi_{12}}}x'_b$$

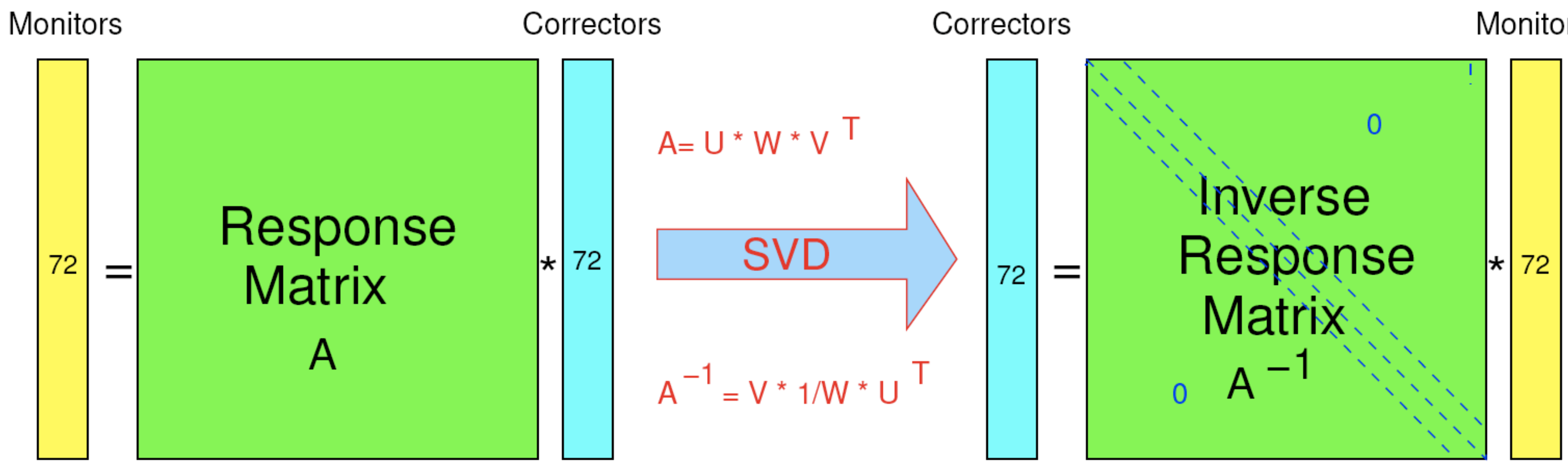
$$\theta_3 = \frac{1}{\sqrt{\beta_3\beta_s}}\frac{\cos\psi_{s4} - \alpha_s \sin\psi_{s4}}{\sin\psi_{34}}x_b - \sqrt{\frac{\beta_s}{\beta_3}\frac{\sin\psi_{s4}}{\sin\psi_{34}}}x'_b$$

$$\theta_4 = \frac{1}{\sqrt{\beta_4\beta_s}}\frac{\cos\psi_{s3} - \alpha_s \sin\psi_{s3}}{\sin\psi_{34}}x_b + \sqrt{\frac{\beta_s}{\beta_4}\frac{\sin\psi_{s3}}{\sin\psi_{34}}}x'_b$$

- **4-bump**: works for any lattice
- Cancels position and angle outside of the bump
- Can be used for aperture scanning

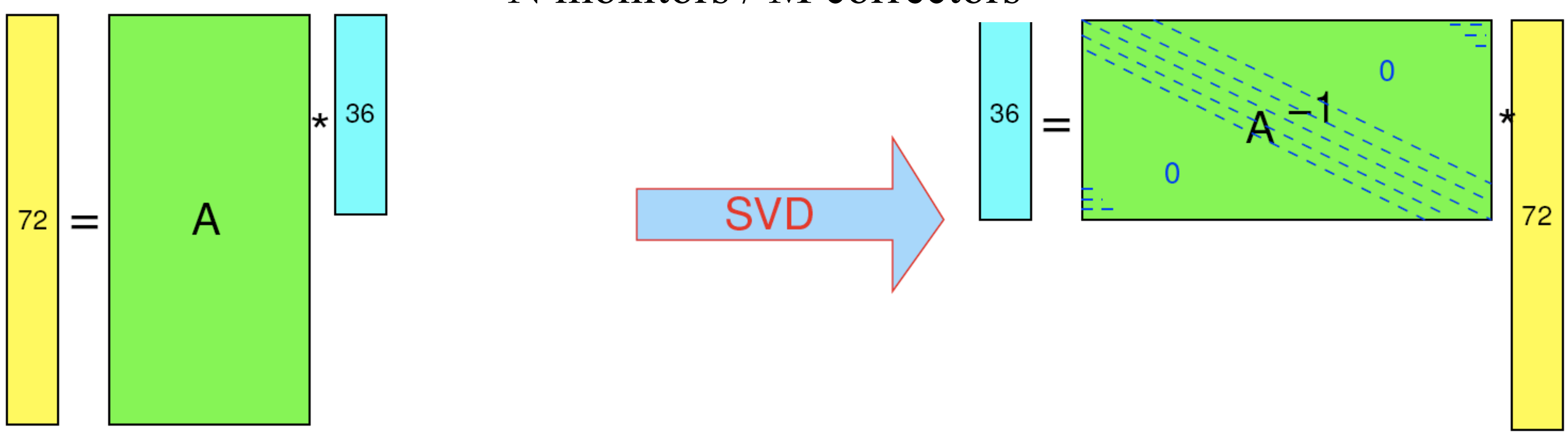
Singular Value Decomposition example

N monitors / N correctors



=> Minimization of the RMS orbit (=0 in case of "Matrix Inversion" using all Eigenvalues)

N monitors / M correctors



=> Minimization of the RMS orbit (monitor averaging)

M. Boege, CAS 2003

Orbit feedback

- Closed orbit stabilization performed using slow and fast orbit feedback system.
- Slow feedback operates every few seconds and uses complete set of BPMs for both planes
- Efficient in correcting distortion due to current decay in magnets or other slow processes
- Fast orbit correction system operates in a wide frequency range (up to 10kHz for the ESRF) correcting distortions induced by quadrupole and girder vibrations.
- Local feedback systems used to damp oscillations in areas where beam stabilization is critical (interaction points, insertion devices)

	β @ BPM [m]	rms orbit [μm]	rms orbit with feedback [μm]
Horizontal	36	5-12	1.2-2.2
Vertical	5.6	1.5-2.5	0.8-1.2

Feedback performance

Summary of integrated rms beam motion (1-100 Hz) with FOFB
and comparison with 10% beam stability target

	FOFB BW	**Horizontal**	**Vertical**
ALS	40 Hz	< 2 μm in H (30 μm)*	< 1 μm in V (2.3 μm)*
APS	60 Hz	< 3.2 μm in H (6 μm)**	< 1.8 μm in V (0.8 μm)**
Diamond	100 Hz	< 0.9 μm in H (12 μm)	< 0.1 μm in V (0.6 μm)
ESRF	100 Hz	< 1.5 μm in H (40 μm)	~ 0.7 μm in V (0.8 μm)
ELETTRA	100 Hz	< 1.1 μm in H (24 μm)	< 0.7 μm in V (1.5 μm)
SLS	100 Hz	< 0.5 μm in H (9.7 μm)	< 0.25 μm in V (0.3 μm)
SPEAR3	60Hz	~ 1 μm in H (30 μm)	~ 1 μm in V (0.8 μm)

* up to 500 Hz

** up to 200 Hz

Trends on Orbit Feedback

- restriction of tolerances w.r.t. to beam size and divergence
- higher frequencies ranges
- integration of XBPMs
- feedback on beamlines components

R. Bartolini, LER2010

Gradient error and optics distortion

- Optics functions perturbation can induce aperture restrictions
- Tune perturbation can lead to dynamic aperture loss
- Broken super-periodicity -> excitation of all resonances
- Causes
  - Errors in quadrupole strengths (random and systematic)
  - Injection elements
  - Higher-order multi-pole magnets and errors
- Observables
  - Tune-shift
  - Beta-beating
  - Excitation of integer and half integer resonances

Gradient error

- Consider the transfer matrix for 1-turn

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a gradient error in a quad. In thin element approximation the quad matrix with and without error are

$$m_0 = \begin{pmatrix} 1 & 0 \\ -K_0(s)ds & 1 \end{pmatrix} \text{ and } m = \begin{pmatrix} 1 & 0 \\ -(K_0(s) + \delta K)ds & 1 \end{pmatrix}$$

- The new 1-turn matrix is $\mathcal{M} = m m_0^{-1} \mathcal{M}_0 = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$ which yields

$$\mathcal{M} = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ \delta K ds(\cos(2\pi Q) - \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Consider a new matrix after 1 turn with a new tune $\chi = 2\pi(Q+\delta Q)$

$$\mathcal{M}^\star = \begin{pmatrix} \cos(\chi) + \alpha_0 \sin(\chi) & \beta_0 \sin(\chi) \\ -\gamma_0 \sin(\chi) & \cos(\chi) - \alpha_0 \sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal $\quad \mathrm{Tra}(\mathcal{M}^\star) = \mathrm{Tra}(\mathcal{M})$

  which gives $2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q+\delta Q))$
- Developing the left hand side

$$\cos(2\pi(Q+\delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{2\pi\delta Q}$$

  and finally $4\pi\delta Q = \delta K ds \beta_0$
- For a quadrupole of finite length, we have

$$\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds$$

Gradient error and beta distortion

- Consider the unperturbed transfer matrix for one turn

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \quad \text{with} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

$$m_{12}^\star = (\beta_0 + \delta\beta) \sin(2\pi(Q + \delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

where we used $\sin(2\pi\delta Q) \approx 2\pi\delta Q$ and $\cos(2\pi\delta Q) \approx 1$

Gradient error and beta distortion

- On the other hand

$$a_{12} = \sqrt{\beta_0 \beta(s_1)} \sin\psi, \;\; b_{12} = \sqrt{\beta_0\beta(s_1)} \sin(2\pi Q - \psi)$$

and $m_{12}^{\star} = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12}} - a_{12}b_{12}\delta K ds = m_{12} - a_{12}b_{12}\delta K ds$

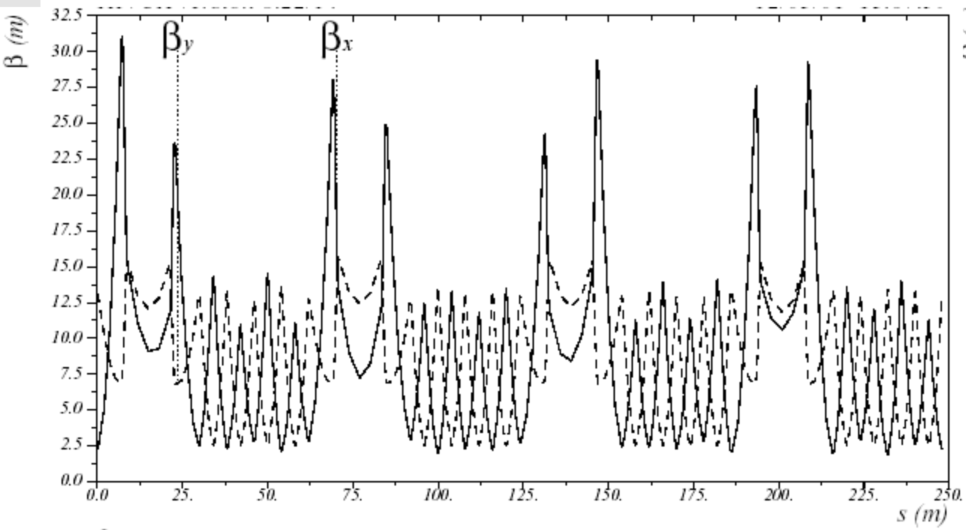
- Equating the two terms

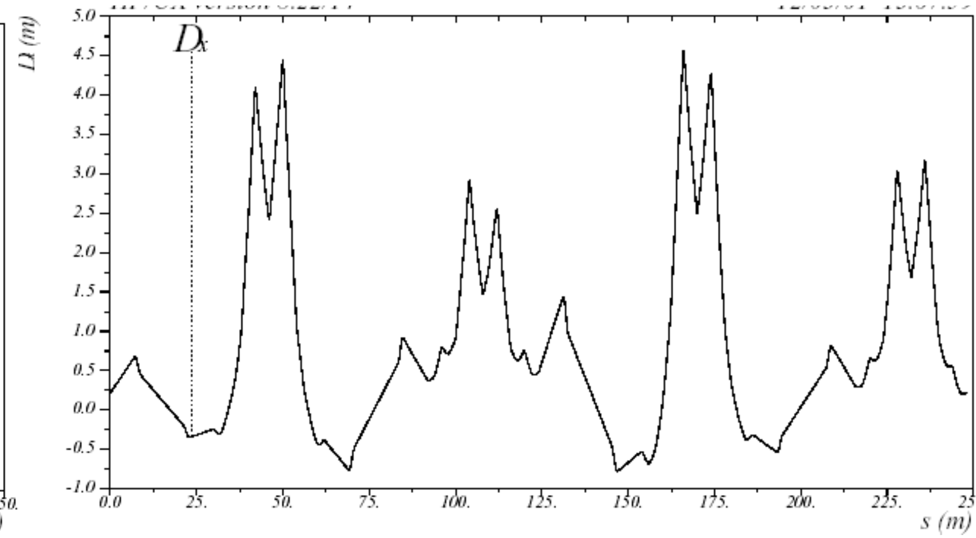
$$\delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q) = -a_{12}b_{12}\delta K ds$$

- Integrating through the quad

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)} \int_{s_1}^{s_1+l} \beta(s)\delta K(s) \cos(2\psi - 2\pi Q) ds$$

Example: Gradient error in the SNS storage ring





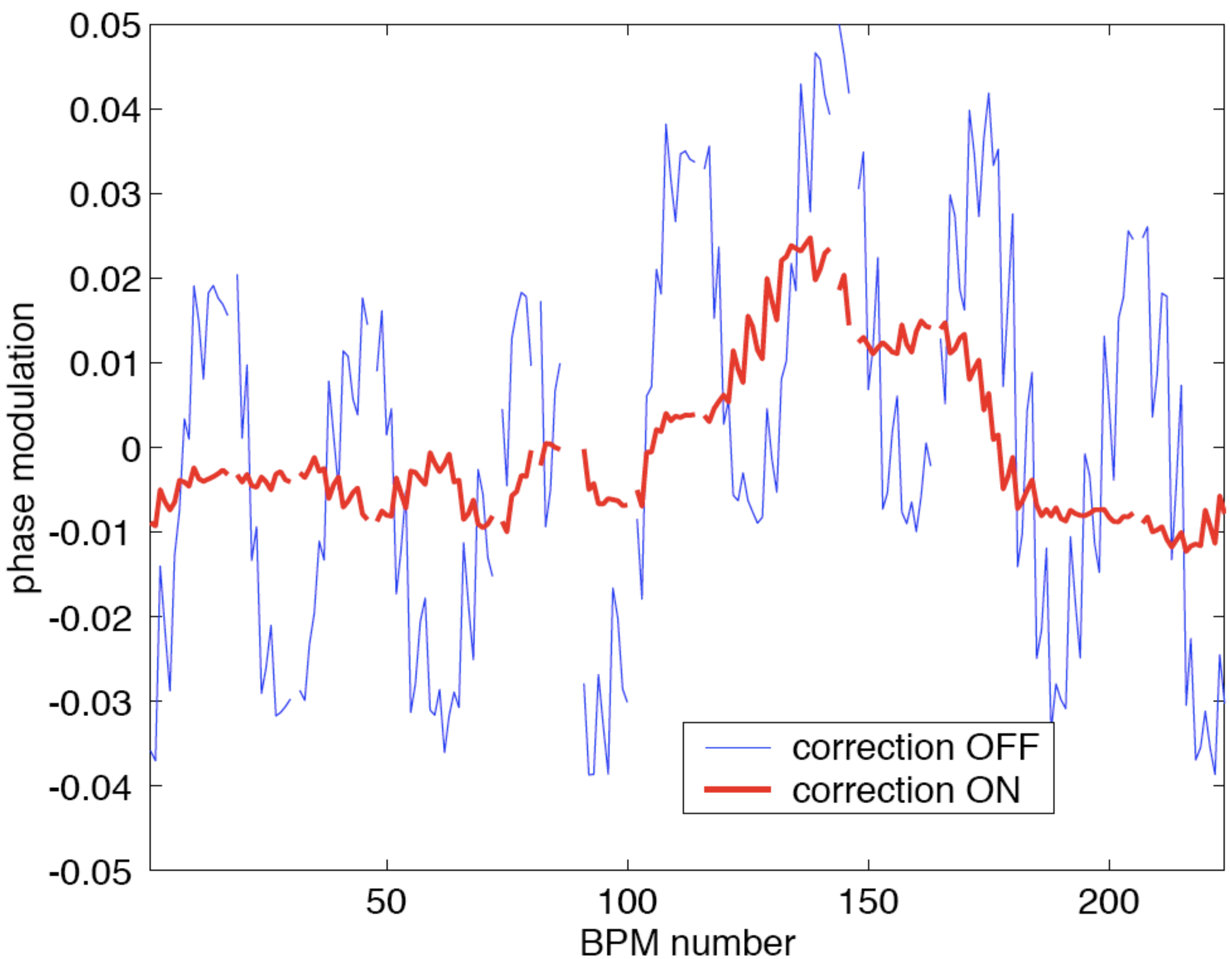
- Consider **18** focusing quads in the SNS ring with **0.01T/m** gradient error. In this location *β*=**12m**. The length of the quads is **0.5m**
- The tune-shift is $\delta Q = \frac{1}{4\pi} 18 \cdot 12 \frac{0.01}{5.6567} 0.5 = 0.015$
- For a random distribution of errors the beta beating is

$$\frac{\delta\beta}{\beta_0}_{\mathrm{rms}} = -\frac{1}{2\sqrt{2}|\sin(2\pi Q)|}\left(\sum_i \delta k_i^2 \beta_i^2\right)^{1/2}$$

- Optics functions beating **> 20%** by putting random errors (1% of the gradient) in high dispersion quads of the SNS ring
- Justifies the choice of corrector strength (trim windings)

Example: Gradient error in the ESRF storage ring

- Consider **128** focusing arc quads in the ESRF storage ring with **0.001T/m** gradient error. In this location ***β*=30m**. The length of the quads is around **1m**



- The tune-shift is

$$\delta Q = \frac{1}{4\pi} 128 \cdot 30 \frac{0.001}{20} 1 = 0.014$$

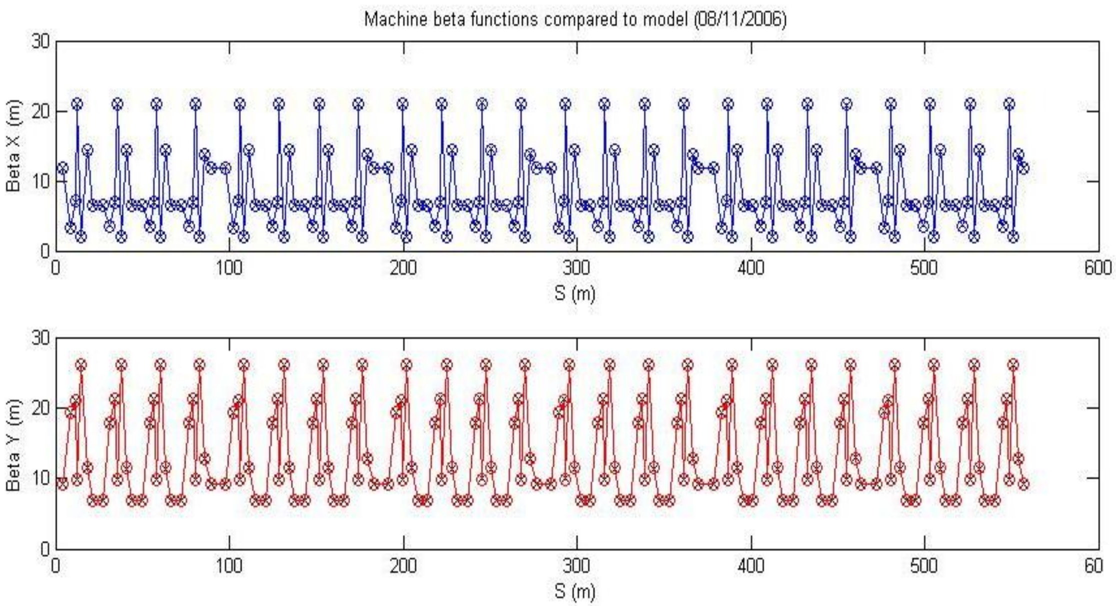
Gradient error correction

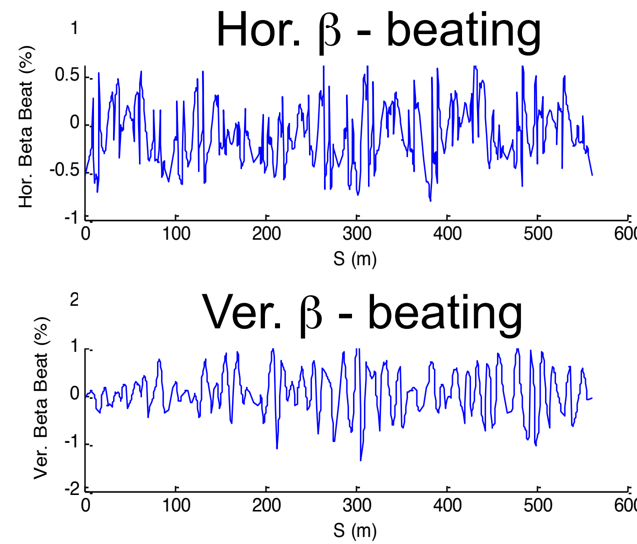
- Windings on the core of the quadrupoles or individual correction magnets (trim windings or quadrupoles)
- Compute tune-shift and optics function beta distortion
- Move working point close to integer and half integer resonance
- Minimize beta wave or quadrupole resonance width with trim windings
- Individual powering of trim windings can provide flexibility and beam based alignment of BPM
- Modern methods of response matrix analysis (LOCO) can fit optics model to real machine and correct optics distortion

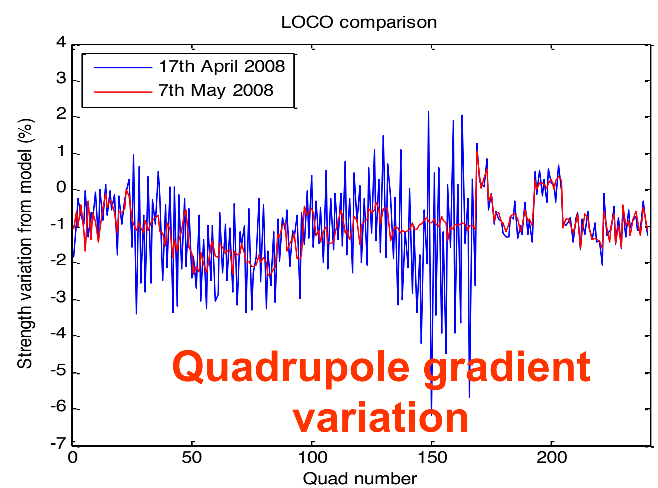
Linear Optics from Closed Orbit

R. Bartolini, LER2010

J. Safranek et al.







Modified version of LOCO with constraints on gradient variations **(see [ICFA Newsl, Dec' 07])**

β - beating reduced to 0.4% rms

Quadrupole variation reduced to 2%

Results compatible with mag. meas. and calibrations

LOCO allowed remarkable progress with the correct implementation of the linear optics

4x4 Matrices

- Combine the matrices for each plane

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

to get a total 4x4 matrix

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

Uncoupled motion

Linear coupling

- Betatron motion is coupled in the presence of skew quadrupoles
- The field is $(B_x, B_y) = k_s(x, y)$ and Hill's equations are coupled
- Motion still linear with two new eigen-mode tunes, which are always split. In the case of a thin skew quad:

$$\delta Q \propto |k_s| \sqrt{\beta_x \beta_y}$$

- Coupling coefficients

$$|C_{\pm}| = \left| \frac{1}{2\pi} \oint ds k_s(s) \sqrt{\beta_x(s)\beta_y(s)} e^{i(\psi_x \pm \psi_y - (Q_x \pm Q_y - q_{\pm}) 2\pi s / C)} \right|$$

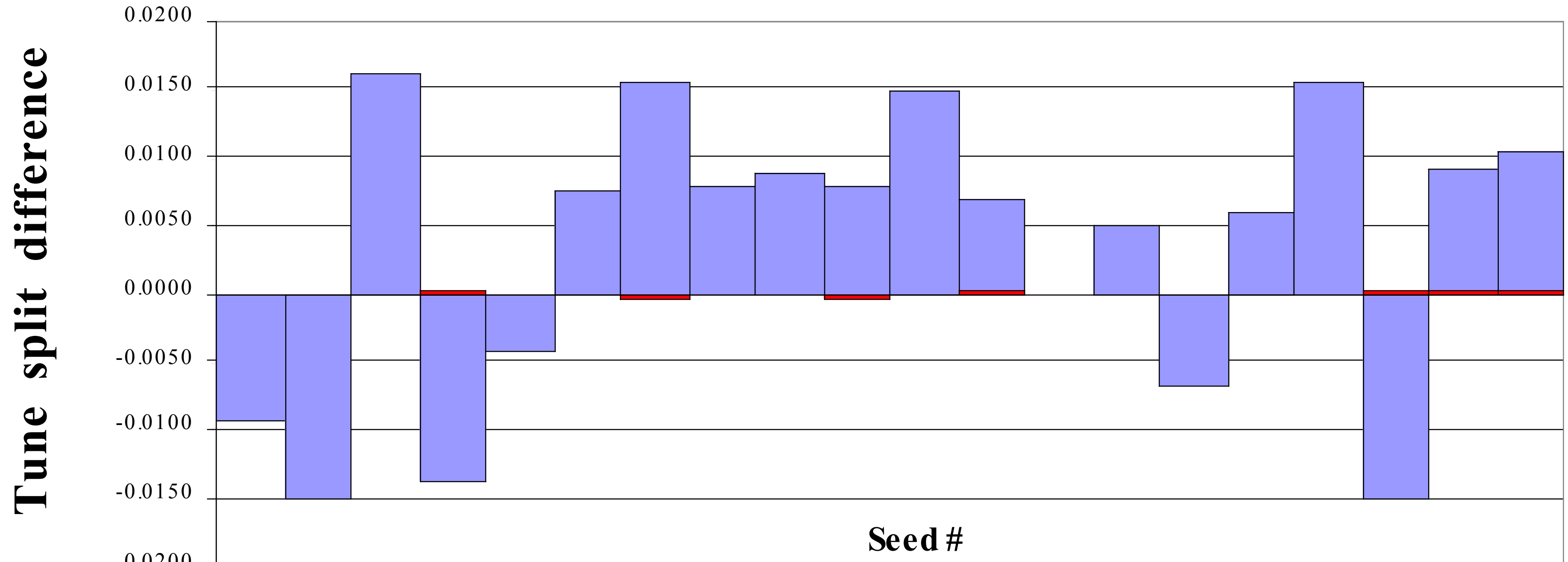
- As motion is coupled, vertical dispersion and optics function distortion appears
- Causes:
  - Random rolls in quadrupoles
  - Skew quadrupole errors
  - Vertical off-sets in sextupoles

Linear coupling correction

- Introduce skew quadrupole correctors
- Correct globally/locally coupling coefficient (or resonance driving term)
- Correct optics distortion (especially vertical dispersion)
- Move working point close to coupling resonances and repeat
- Correction especially important for flat beams
- Note that (vertical) orbit correction may be critical for reducing coupling

Example: Coupling correction for the SNS ring

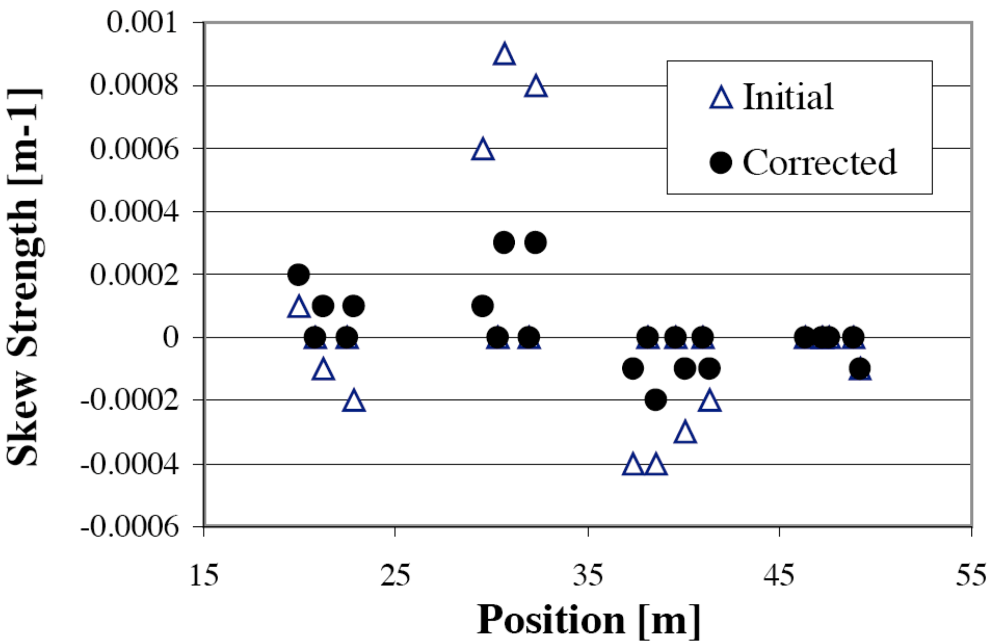
- Local decoupling by super period using 16 skew quadrupole correctors
- Results of Q_x=6.23 Q_y=6.20 after a **2mrad** quad roll
- Additional 8 correctors used to compensate vertical dispersion

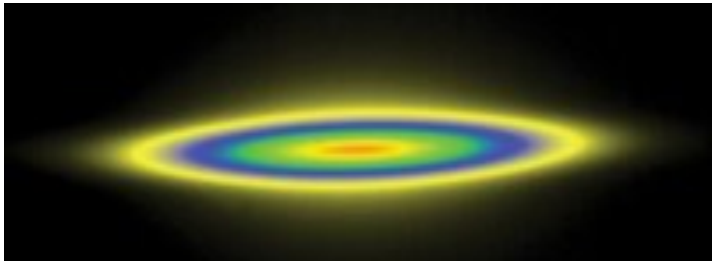


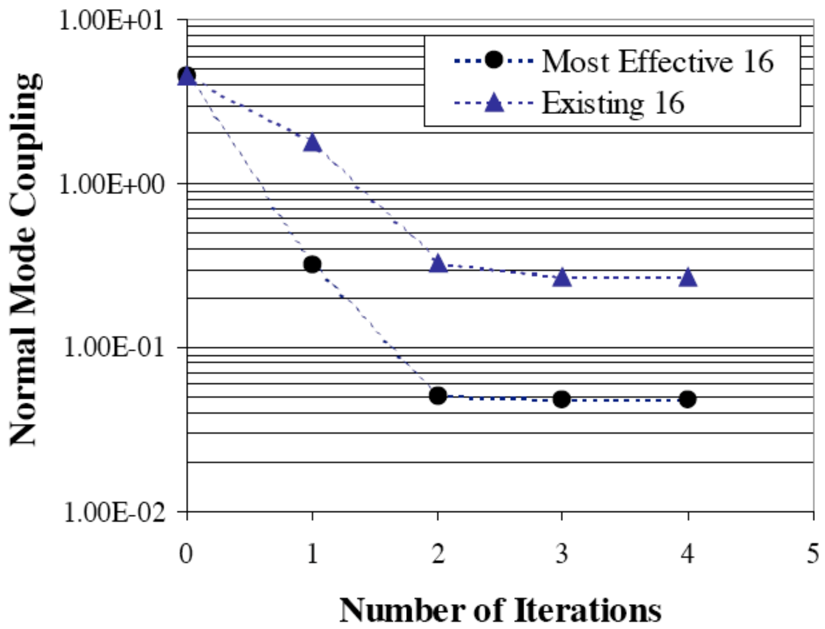
Seed #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Before Correction	-0.009	-0.014	0.016	-0.013	-0.004	0.007	0.015	0.008	0.008	0.007	0.014	0.006	0.000	0.005	-0.006	0.006	0.015	-0.015	0.009	0.010
After correction	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000

Example: Coupling correction for the ESRF ring

- Local decoupling using 16 skew quadrupole correctors and coupled response matrix reconstruction
- Achieved correction of below 0.25% reaching vertical emittance of below 4pm

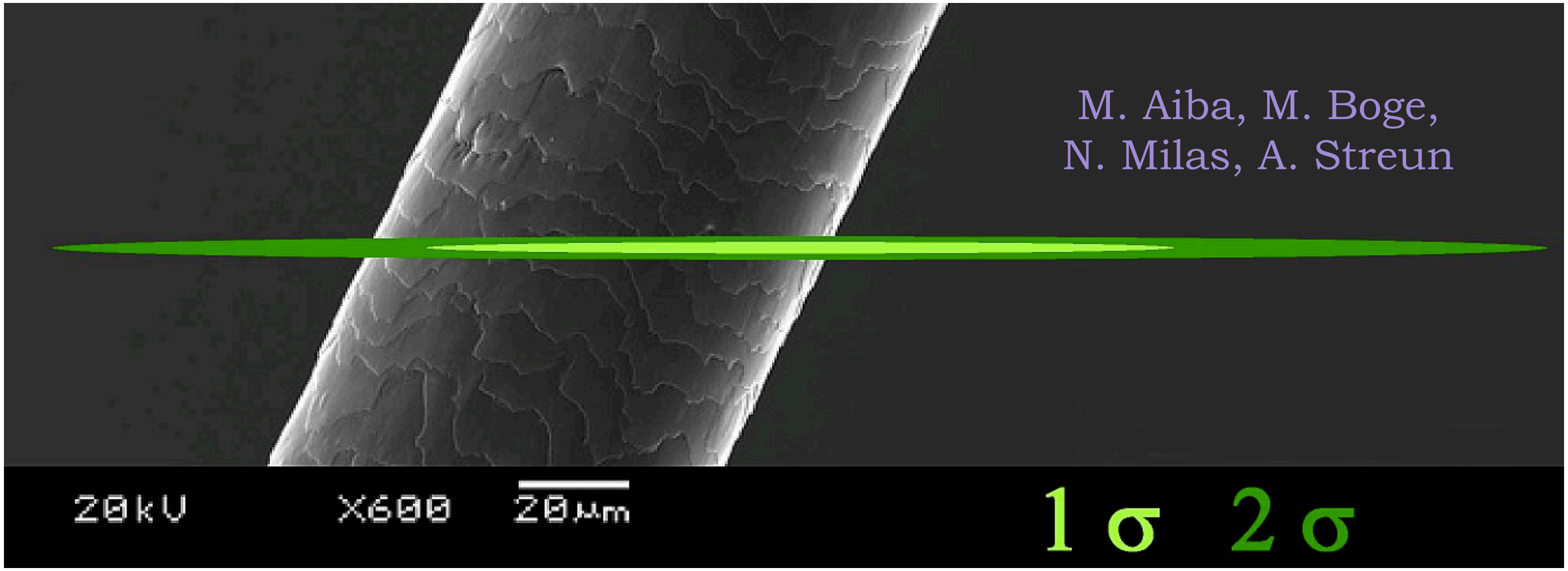






R. Nagaoka, EPAC 2000

Vertical emittance record @ PSI



- Vertical emittance reduced to a minimum value of **0.9±0.4pm**
- Achieved by carefull re-alignment campaign and different methods of coupling suppression using 36 skew quadrupoles (combination of response matrix based correction and random walk optimisation)
- Performance of emittance monitor had to be further stretched to get beam profile data at a size of around 3-4µm

Chromaticity

- Linear equations of motion depend on the energy (term proportional to dispersion)
- Chromaticity is defined as: $\xi_{x,y} = \frac{\delta Q_{x,y}}{\delta p/p}$
- Recall that the gradient is $k = \frac{G}{B\rho} = \frac{eG}{p} \rightarrow \frac{\delta k}{k} = \mp \frac{\delta p}{p}$
- This leads to dependence of tunes and optics function on energy
- For a linear lattice the tune shift is:

$$\delta Q_{x,y} = \frac{1}{4\pi} \oint \beta_{x,y} \delta k(s) ds = -\frac{1}{4\pi} \frac{\delta p}{p} \oint \beta_{x,y} k(s) ds$$

- So the **natural** chromaticity is:

$$\xi_{x,y} = -\frac{1}{4\pi} \oint \beta_{x,y} k(s) ds$$

- Sometimes the chromaticity is quoted as $\overline{\xi_{x,y}} = \frac{\xi_{x,y}}{Q_{x,y}}$

Example: Chromaticity in the SNS ring

- In the SNS ring, the natural chromaticity is –7.
- Consider that momentum spread $\frac{\delta P}{P} = \pm 1\%$
- The tune-shift for off-momentum particles is

$$\delta Q_{x,y} = \xi_{x,y} \frac{\delta P}{P} = \pm 0.07$$

- In order to correct chromaticity introduce particles which can focus off-momentum particle

⬇

Sextupoles

Chromaticity from sextupoles

- The sextupole field component in the *x*-plane is: $B_y = \frac{S}{2}x^2$
- In an area with non-zero dispersion $x = x_0 + D\frac{\delta P}{P}$
- Than the field is

$$B_y = \frac{S}{2}x_0^2 + \underbrace{SD\frac{\delta P}{P}x_0}_{\text{quadrupole}} + \underbrace{\frac{S}{2}D^2\frac{\delta P}{P}^2}_{\text{dipole}}$$

- Sextupoles introduce an equivalent focusing correction

$$\delta k = SD\frac{\delta P}{P}$$

- The sextupole induced chromaticity is

$$\xi_{x,y}^{S} = -\frac{1}{4\pi}\oint \mp\beta_{x,y}(s)S(s)D_x(s)ds$$

- The total chromaticity is the sum of the natural and sextupole induced chromaticity

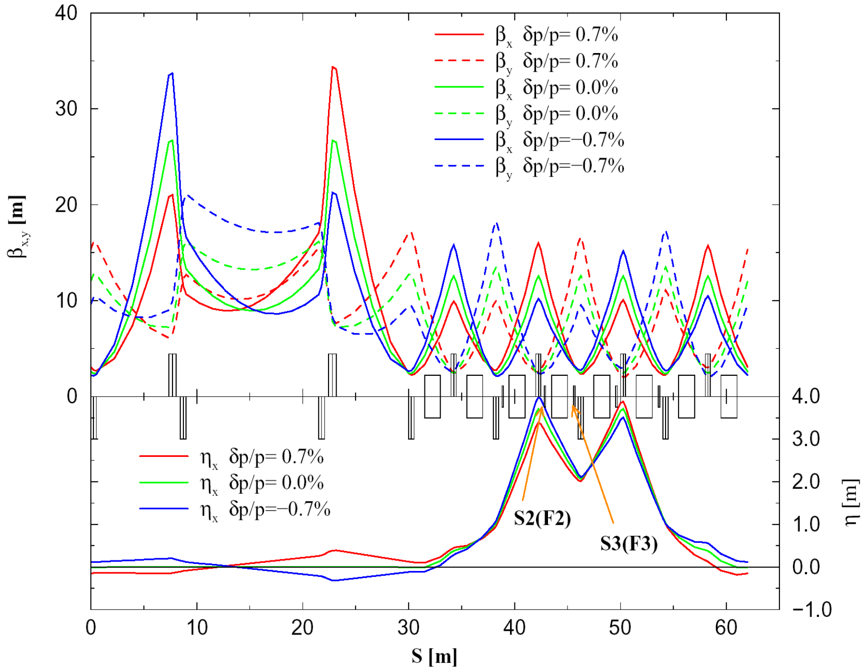
$$\xi_{x,y}^{\text{tot}} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)\left(k(s) \mp S(s)D_x(s)\right)ds$$

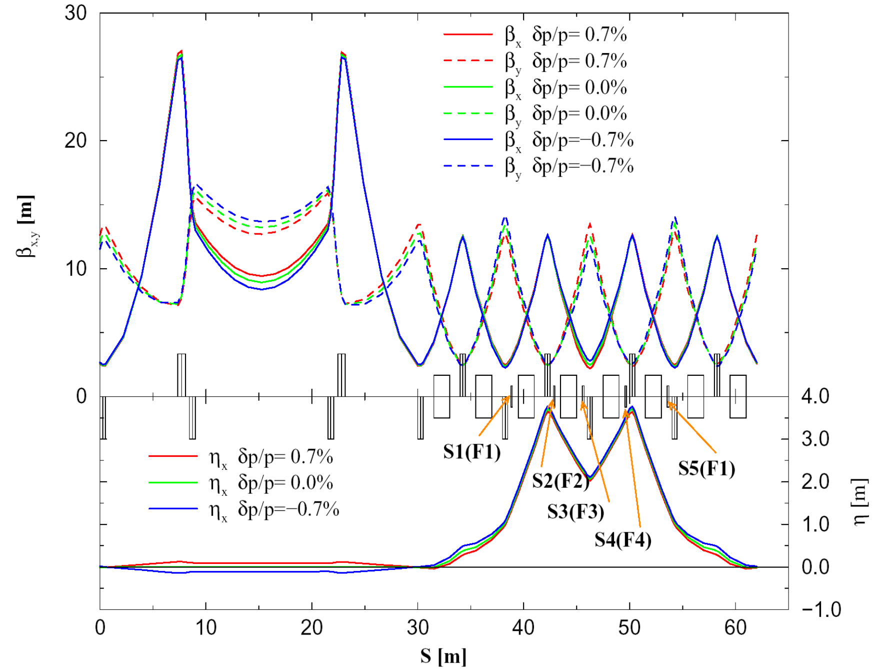
Chromaticity correction

- Introduce sextupoles in high-dispersion areas
- Tune them to achieve desired chromaticity
- Two families are able to control horizontal and vertical chromaticity
- Sextupoles introduce non-linear fields (chaotic motion)
- Sextupoles introduce tune-shift with amplitude
- Example:
  - The SNS ring has natural chromaticity of **–7**
  - Placing two sextupoles of length **0.3m** in locations where β=**12m**, and the dispersion D=**4m**
  - For getting **0** chromaticity, their strength should be

$$S = \frac{7 \cdot 4\pi}{12 \cdot 4 \cdot 2 \cdot 0.3} \approx 3\mathrm{m}^{-3}$$ or a gradient of **17.3 T/m²**

Two vs. four families for chromaticity correction





- Two families of sextupoles not enough for correcting off-momentum optics functions' distortion and second order chromaticity
- Solutions:
  - Place sextupoles accordingly to eliminate second order effects (difficult)
  - Use more families (4 in the case of of the SNS ring)
- Large optics function distortion for momentum spreads of **±0.7%**,when using only two families of sextupoles
- Absolute correction of optics beating with four families

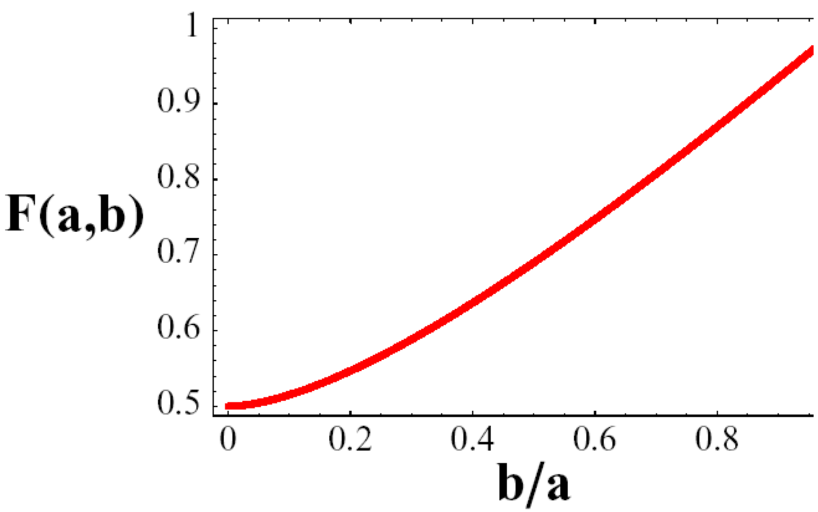
Eddy current sextupole component

$$\xi_{x,y}^{\text{eddy}} = \pm \frac{1}{4\pi} \oint S^{\text{eddy}}(s,t) D_x(s) \beta_{x,y}(s) ds$$

Sextupole component due to Eddy currents in an elliptic vacuum chamber of a pulsing dipole

$$S^{\text{eddy}}(t) = \frac{1}{B\rho} \frac{d^2 B_y}{dx^2} = \frac{1}{B\rho} \frac{\mu_0 \sigma_c t \dot{B}_y}{h} F(a,b)$$

with $F(a,b) = \int_0^{\pi/2} \sin\phi \sqrt{\cos^2\phi + (b/a)^2 \sin^2\phi}\, d\phi = 1/2 \left[1 + \frac{b^2 \text{arcsinh}(\sqrt{a^2 - b^2}/b)}{a\sqrt{a^2 - b^2}} \right]$



Taking into account

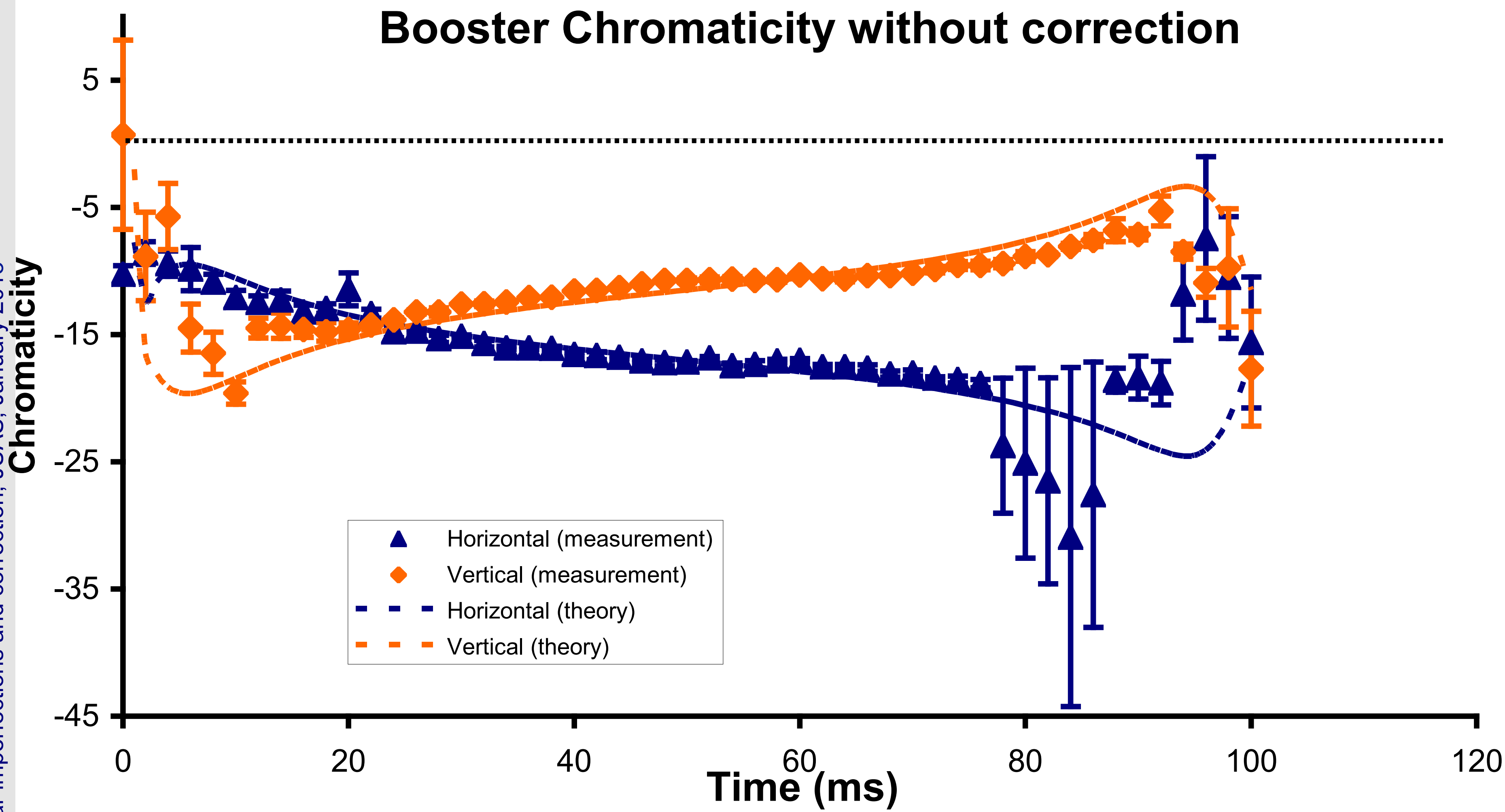
$$B_y(t) = \frac{B_{\max}}{1 + a_E} (a_E - \cos(\omega t))$$

with

$$a_E = \frac{E_{\max} + E_{\min}}{E_{\max} - E_{\min}}$$

we get $S^{\text{eddy}}(t) = \frac{\mu_0\ \sigma_c\ t\ \omega}{h\ \rho} \frac{\sin(\omega t)}{a_E - \cos(\omega t)} F(a,b)$

Example: ESRF booster chromaticity



Problems

1) A **proton** ring with kinetic energy of **1GeV** and a **circumference** of **248m** has **18, 1m-long** focusing quads with **gradient** of **5T/m**. In one of the quads, the horizontal and vertical **beta** function is of **12m** and **2m** respectively. The **rms beta** function in both planes on the focusing quads is **8m**. With a **horizontal tune** of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on the single focusing quad given by **horizontal** and by **vertical** misalignments of **1mm** in all the quads. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?

2) Three correctors are placed at locations with phase advance of **π/4** between them and beta functions of **12**, **2** and **12m**. How are the corrector kicks related to each other in order to achieve a closed 3-bump.

3) Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **19.4 T/m**, with a horizontal and vertical **beta** of **108m** and **18m** in the focusing quads which are **18m** and **108m** for the defocusing ones. Find the tune change for systematic gradient errors of **1%** in the focusing and **0.5%** in the defocusing quads. What is the chromaticity of the machine?

4) Derive an expression for the resulting magnetic field when a normal sextupole with field $\mathbf{B = S/2\ x^2}$ is displaced by $\mathbf{\delta x}$ from its center position. At what type of fields correspond the resulting components? Do the same for an octupole with field $\mathbf{B = O/3\ x^3}$. What is the leading order multi-pole field error when displacing a general **2n**-pole magnet?