



Physics of Synchrotron Radiation

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- Synchrotron light and radiation
- Typical storage ring - parameters
- Radiation power
- Characteristics of synchrotron radiation
 - Time compression
 - Angular collimation
 - Radiation spectrum
 - Flux and brilliance

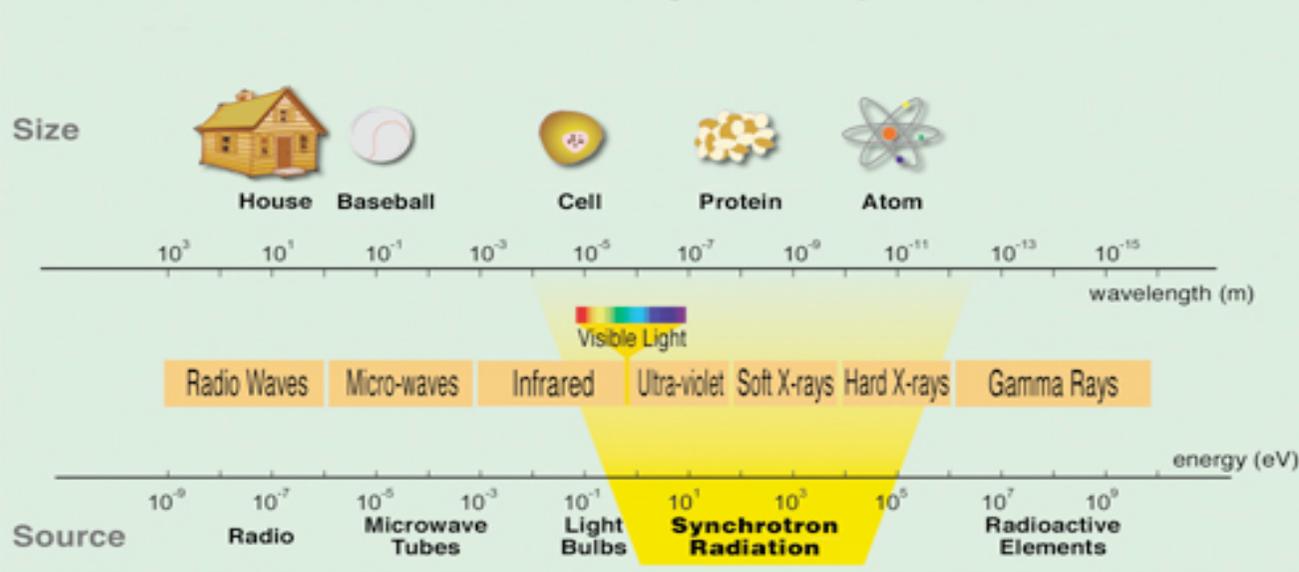


Röntgen, 1895



Physics of Synchrotron Radiation, USPAS, January 2008

The Electromagnetic Spectrum



J. P. Blewett, *Phys. Rev.* **69**, 87 (1946);
 F. R. Elder, R. V. Langmuir, A. M. Gurewitsch, H. C. Pollock, *Phys. Rev.* **71**, 827 (1947)



The Crab nebula is the expanding remains of a star that was seen to explode by Chinese astronomers in the year 1054AD.

At the heart of the nebula is a rapidly-spinning neutron star, a pulsar, and it powers the strongly polarised bluish 'synchrotron' nebula.



Why X-rays are important



18 Nobel Prizes Based on X-ray Work

Chemistry

1936: Peter Debye

1962: Max Perutz and Sir John
Kendrew

1976 William Lipscomb

1985 Herbert Hauptman and Jerome
Karle

1988 Johann Deisenhofer, Robert
Huber and Hartmut Michel

1997 Paul D. Boyer and John E.
Walker

2003 Peter Agre and Roderick
Mackinnon

Physics

1901 Wilhem Rontgen

1914 Max von Laue

1915 Sir William Bragg and son

1917 Charles Barkla

1924 Karl Siegbahn

1927 Arthur Compton

1981 Kai Siegbahn

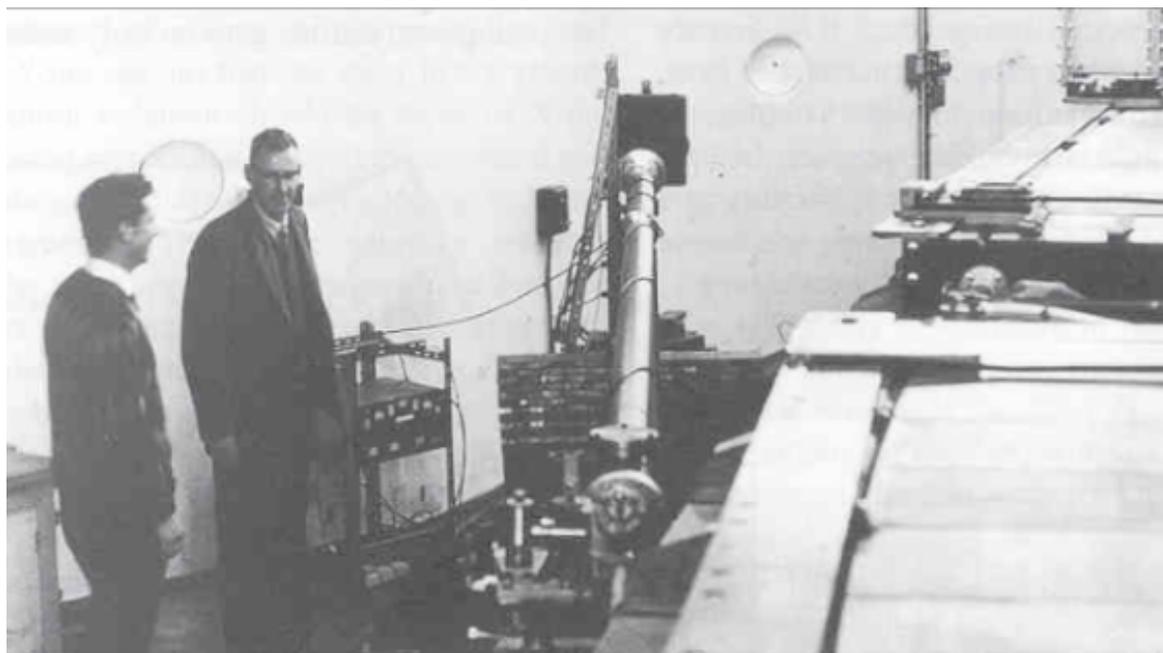
Medicine

1946 Hermann Muller

1962 Frances Crick, James Watson
and Maurice Wilkins

1979 Alan Cormack and Godfrey
Hounsfield

- 1st Generation SR sources
- Electron synchrotrons start to be built for high energy physics use (rapidly cycling accelerators not Storage Rings!)
- Interest from other physicists in using the “waste” SR
- First users are parasitic



The first beamline on NINA at Daresbury constructed in 1966/67 by Manchester University

NINA was a 6GeV electron synchrotron devoted to the study of particle physics

- 2nd Generation SR sources
- Purpose built accelerators start to be built – late 70's
- First users ~1980 (at SRS, Daresbury)
- Based primarily upon bending magnet radiation

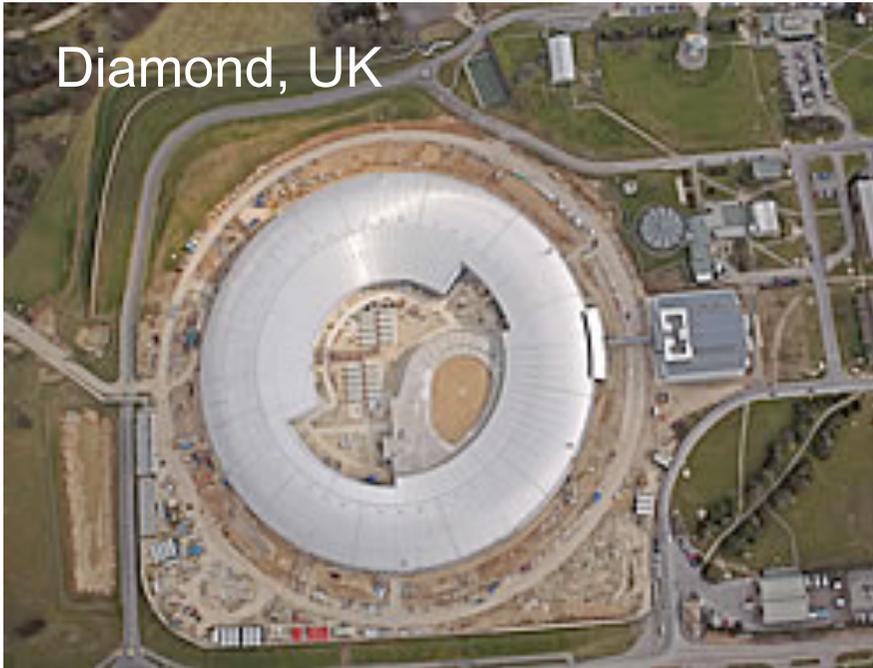


The VUV ring at Brookhaven in 1980 before the beamlines are fitted

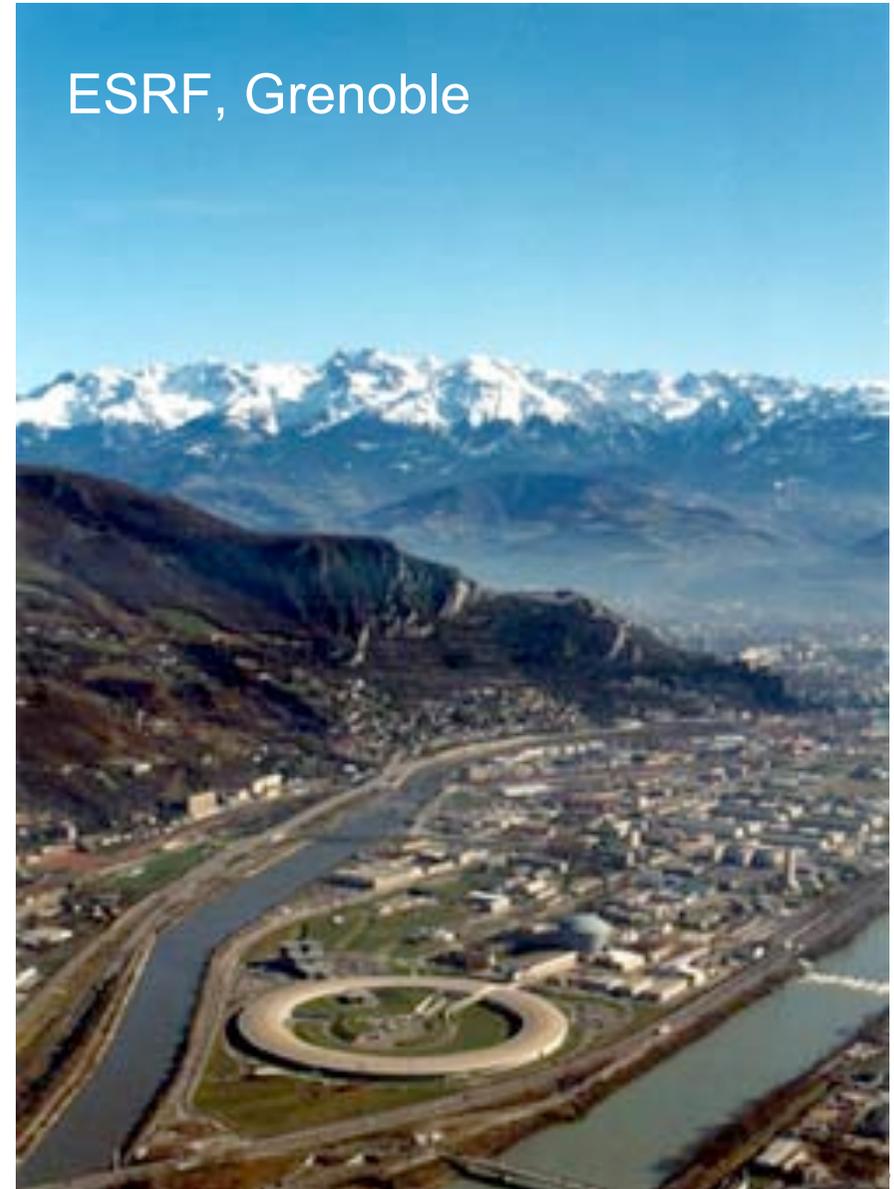
Not much room for undulators!

- 3rd Generation SR sources
- Primary light source is the undulator
- First built in the late 80's/early 90's
- First users ~1994

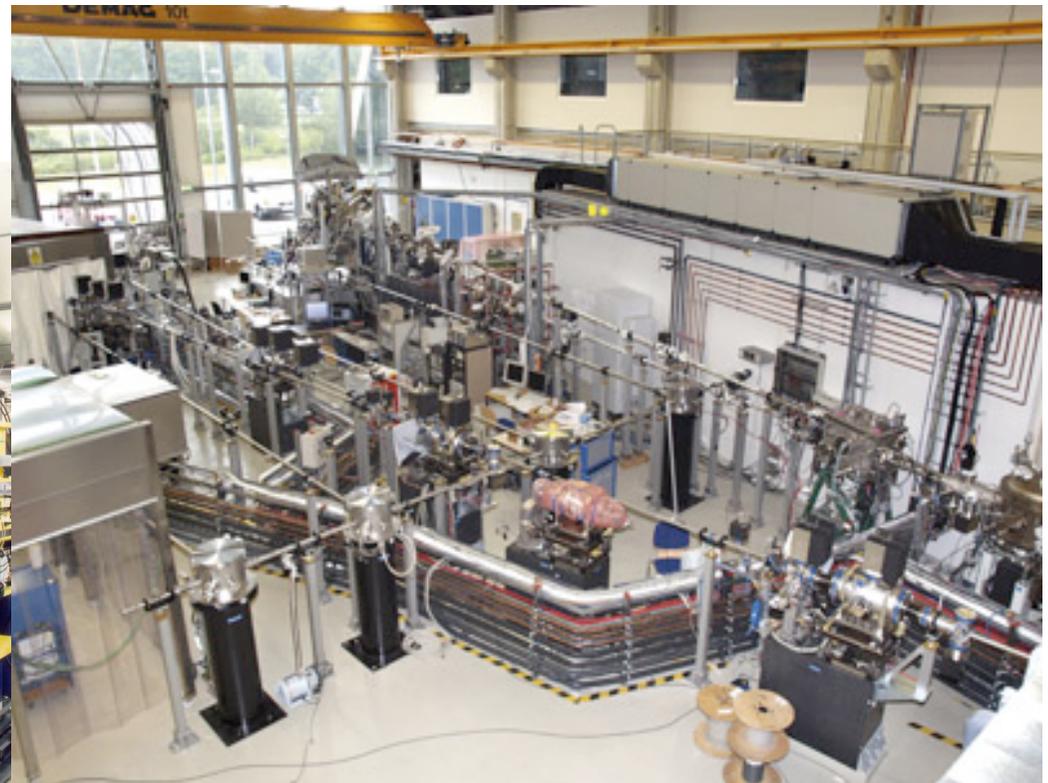
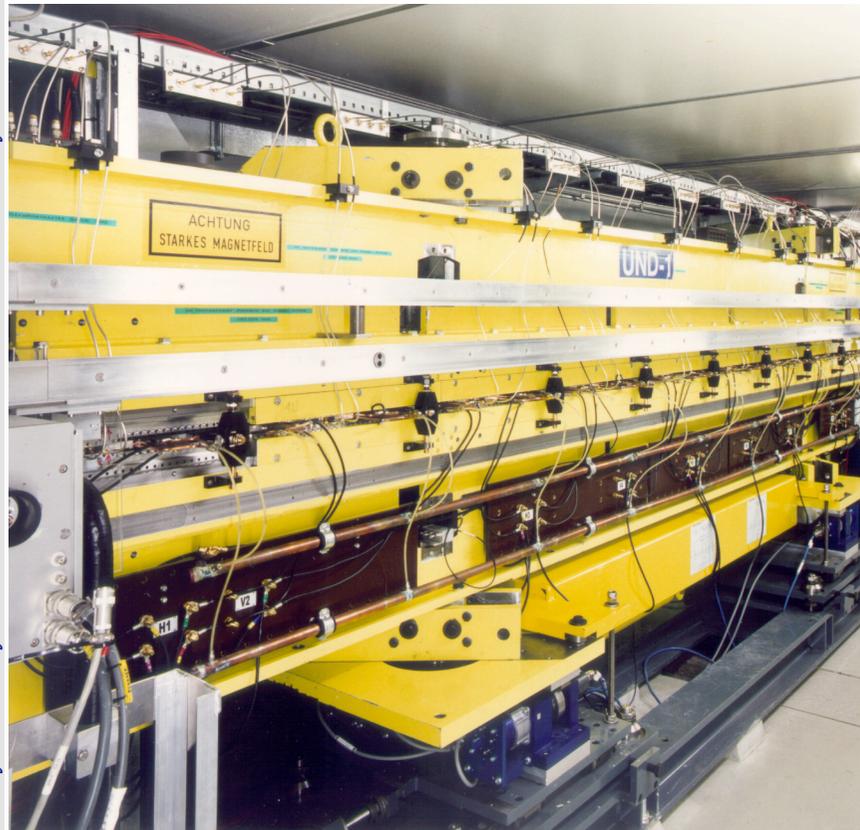
Diamond, UK



ESRF, Grenoble

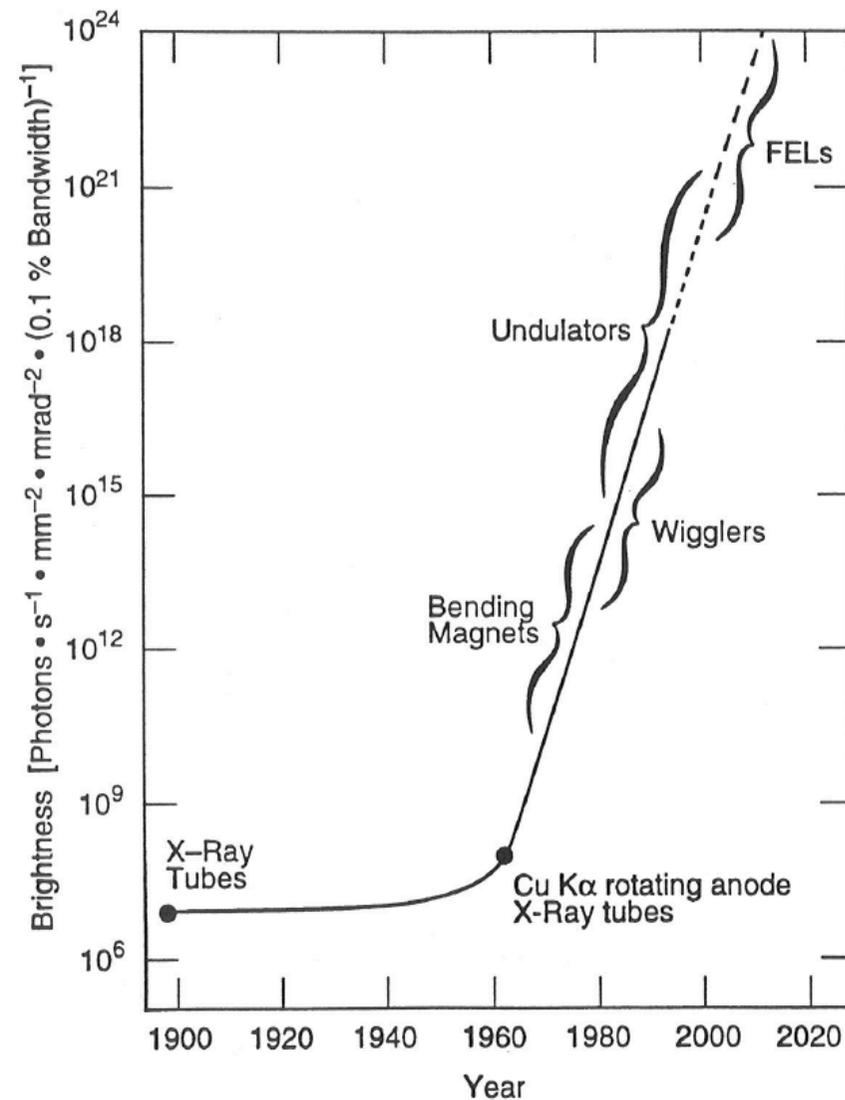


- 4th Generation SR sources
- Primary light source is the single pass Free Electron Laser
- First built ~2000
- First users ~2006



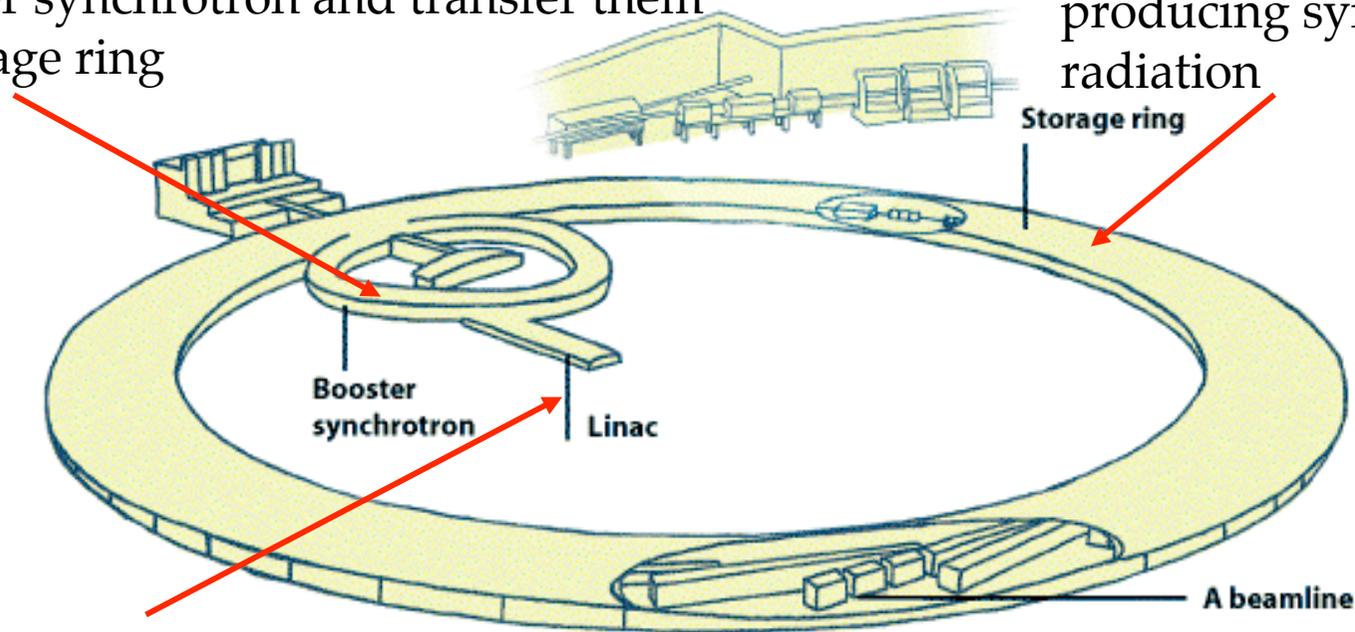
FLASH FEL facility at
DESY

- **X-ray tubes** (early 20th century)
- **1st generation:** originally build for high-energy physics experiments and synchrotron radiation programs used parasitically
- **2nd generation:** dedicated synchrotron sources based on bending magnets
- **3rd generation:** synchrotron radiation is produced in undulators and wigglers
- **4th generation:** free electron lasers



Accelerate electrons up to a few GeV in a few msec in the booster synchrotron and transfer them into the storage ring

Electrons are getting accumulated up to a high current (a few hundred mA) in the storage ring and they circulate freely producing synchrotron radiation



Produce electron in a thermionic gun, accelerate them up to a few MeV in a linac and transfer them into a booster

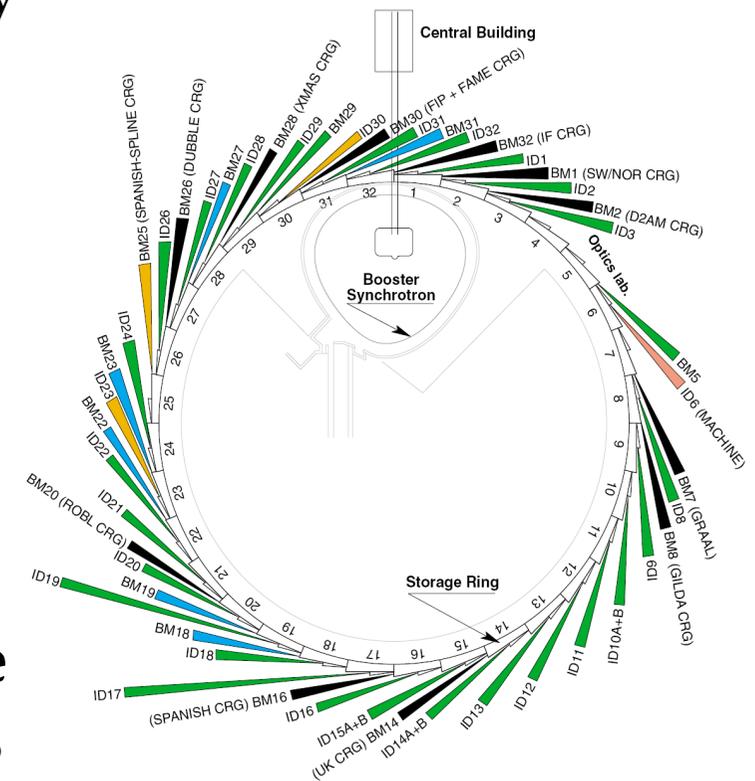
Procedure repeated periodically, depending on the beam lifetime



A typical storage ring – the ESRF



- The **first and most brilliant 3rd** generation light source in Europe
- **50 beam lines** collecting X-rays from insertion devices and bending magnets
- **3500 users/year** from **14 member countries** carrying X-ray spectroscopy experiments for material science, chemistry, biology, medicine, earth sciences, archeology, etc.
- The machine comprises an **e⁻ linac**, a **300m-booster** and an **844m-storage ring**
- The storage ring has a record **availability of 98%** with a **mean-time between failures** of more than **2 days**

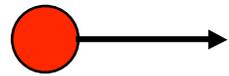




ESRF parameters



Energy	GeV	6.03
Maximum Current	mA	200
Horizontal Emittance	nm	4
Vertical Emittance (*minimum achieved)	nm	0.025 (0.010*)
Coupling (*minimum achieved)	%	0.6 (0.25*)
Revolution frequency	kHz	355
Number of bunches		1 to 992
Time between bunches	ns	2816 to 2.82



$$\mathbf{p} = m_0 \mathbf{v}$$

$$v \ll c$$

$$P_s = \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3} \left(\frac{d\mathbf{p}}{dt} \right)^2$$

Larmor Power radiated by non-relativistic particles is very small

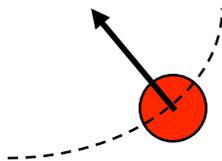


$$\mathbf{p} = \gamma m_0 \mathbf{v}$$

$$v \approx c$$

$$P_s = \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3} \left(\frac{dp}{dt} \right)^2$$

Power radiated by relativistic particle in linear accelerator is negligible



$$P_s = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho^2}$$

Power radiated by relativistic particle in circular accelerator is very strong ([Liénard, 1898](#))

■ “Electric and Magnetic Field produced by an electric charge concentrated at a point and travelling on an arbitrary path”
 Prophetically published in the french journal “The Electric Light”

L'Éclairage Électrique
 REVUE HEBDOMADAIRE D'ÉLECTRICITÉ

DIRECTION SCIENTIFIQUE

A. CORNU, Professeur à l'École Polytechnique, Membre de l'Institut. — A. DARSONVAL, Professeur au Collège de France, Membre de l'Institut. — G. LIFFMANN, Professeur à la Sorbonne, Membre de l'Institut. — D. MONNIE, Professeur à l'École centrale des Arts et Manufactures. — H. POINCARÉ, Professeur à la Sorbonne, Membre de l'Institut. — A. POTIER, Professeur à l'École des Mines, Membre de l'Institut. — J. BLONDIN, Professeur agrégé de l'Université.

CHAMP ÉLECTRIQUE ET MAGNÉTIQUE

PRODUIT PAR UNE CHARGE ÉLECTRIQUE CONCENTRÉE EN UN POINT ET ANIMÉE D'UN MOUVEMENT QUELCONQUE

Admettons qu'une masse électrique en mouvement de densité ρ et de vitesse w en chaque point produit le même champ qu'un courant de conduction d'intensité ap . En conservant les notations d'un précédent article (1) nous obtiendrons pour déterminer le champ, les équations

$$\frac{1}{4\pi} \left(\frac{d^2x}{dt^2} - \frac{d^2y}{dt^2} \right) = \rho u_x + \frac{d\rho}{dt} \quad (1)$$

$$V^2 \left(\frac{d^2x}{dt^2} - \frac{d^2y}{dt^2} \right) = -\frac{1}{4\pi} \frac{d\rho}{dt} \quad (2)$$

avec les analogues déduites par permutation tournante et en outre les suivantes

$$\rho = \left(\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} + \frac{d^2z}{dt^2} \right) \quad (3)$$

$$\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} + \frac{d^2z}{dt^2} = \rho \quad (4)$$

De ce système d'équations on déduit facilement les relations

$$\left(V^2 - \frac{d^2}{dt^2} \right) \rho = V^2 \frac{d^2\rho}{dt^2} + \frac{d^2\rho}{dt^2} \quad (5)$$

$$\left(V^2 - \frac{d^2}{dt^2} \right) \rho = 4\pi V^2 \left[\frac{d}{dt} (\rho u_x) - \frac{d}{dt} (\rho u_y) \right] \quad (6)$$

(1) La théorie de Lorenz, *L'Éclairage Électrique*, t. XIV, p. 417, 4, 5, 7, pour les composantes de la force électrostatique et f, g, h , celles du déplacement dans l'éther.

Soient maintenant quatre fonctions ϕ, F, G, H définies par les conditions

$$\left(V^2 - \frac{d^2}{dt^2} \right) \phi = -4\pi V^2 \rho \quad (7)$$

$$\left(V^2 - \frac{d^2}{dt^2} \right) F = -4\pi V^2 \rho u_x \quad (8)$$

$$\left(V^2 - \frac{d^2}{dt^2} \right) G = -4\pi V^2 \rho u_y \quad (9)$$

$$\left(V^2 - \frac{d^2}{dt^2} \right) H = -4\pi V^2 \rho u_z \quad (10)$$

On satisfera aux conditions (5) et (6) en prenant

$$4\pi \rho = -\frac{d^2\phi}{dt^2} - \frac{1}{V^2} \frac{d^2\rho}{dt^2} \quad (11)$$

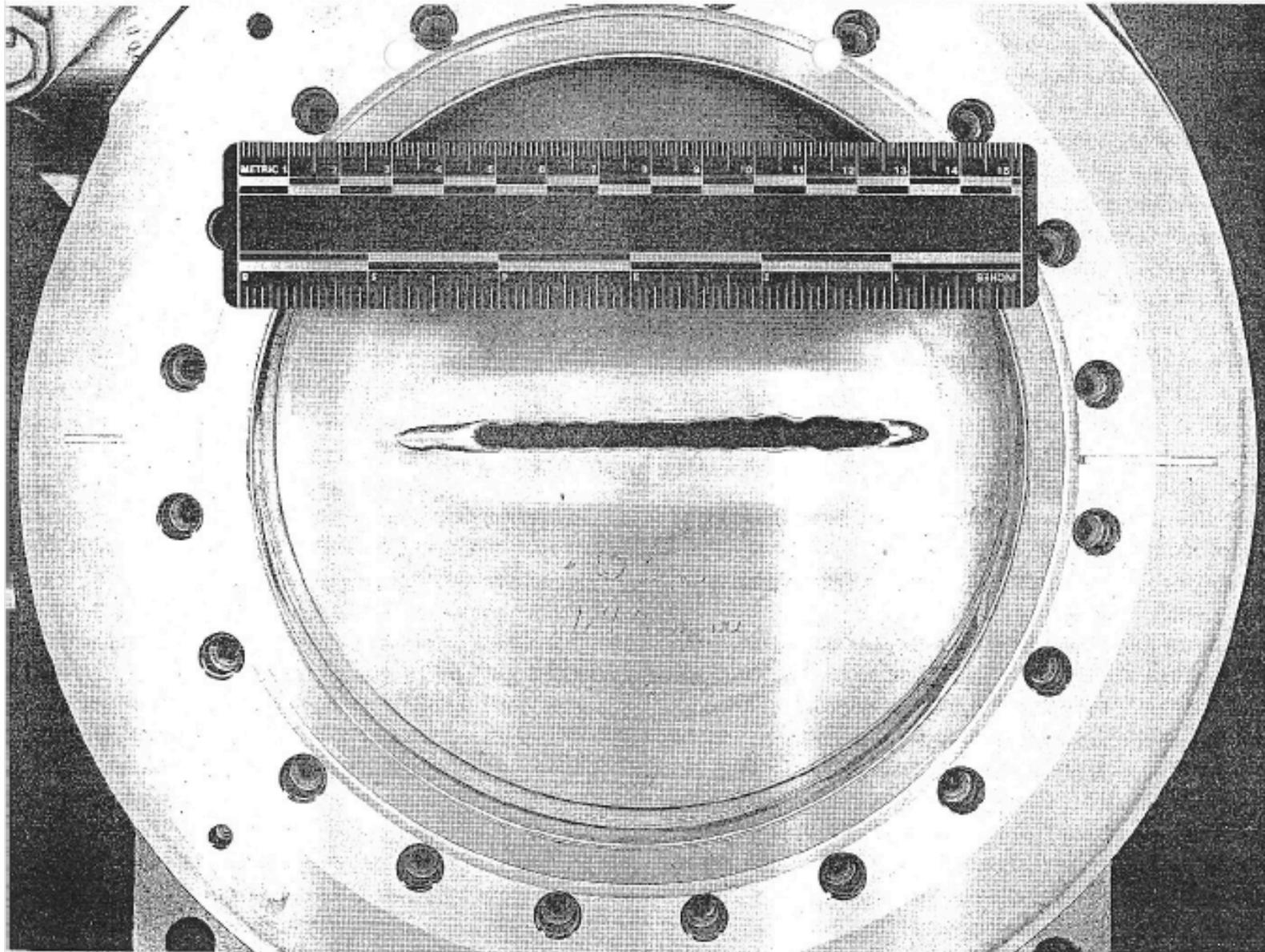
$$u = \frac{dF}{dt} - \frac{dG}{dt} \quad (12)$$

Quant aux équations (7) à (10), pour qu'elles soient satisfaites, il faudra que, en plus de (7) et (10), on ait la condition

$$\frac{d^2\phi}{dt^2} + \frac{d^2F}{dt^2} + \frac{d^2G}{dt^2} + \frac{d^2H}{dt^2} = \rho \quad (13)$$

Occupons-nous d'abord de l'équation (7). On sait que la solution la plus générale est la suivante :

$$\phi = \int \frac{\rho' \left[\frac{r}{r} \left(\frac{r}{r} - \frac{r}{r} \right) \right]}{r} dt \quad (14)$$



$$P_s = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho^2}$$

Power inversely proportional to 4th power of rest **mass** (proton **2000 times** heavier than electron)
 On the other hand, for **multi TeV** hadron colliders (LHC) synchrotron radiation is an important issue (protection with absorbers)

$$\Delta E = \frac{e^2}{3\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho}$$

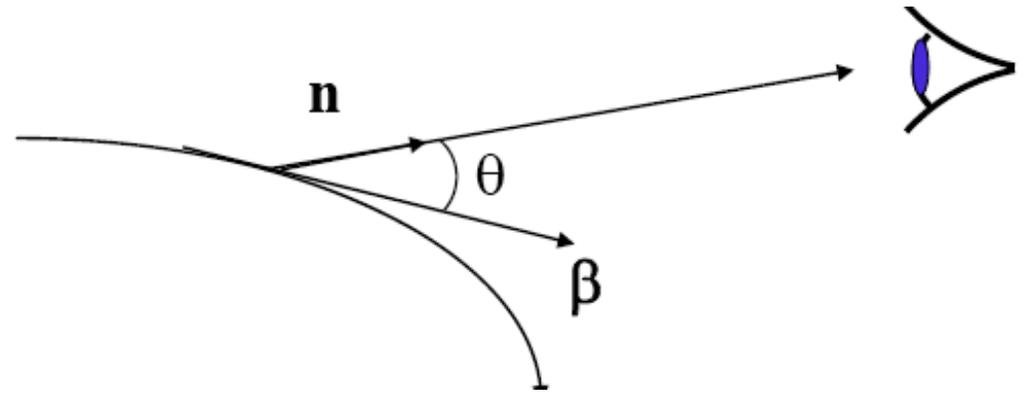
For electrons:

$$\Delta E [keV] = 88.5 \frac{E^4 [GeV^4]}{\rho [m]}$$

By integrating around one revolution we get the **energy loss per turn**. For the ESRF is around 5 MeV/turn. On the other hand, for LEP II (**120 GeV**) it was 6 GeV/turn, i.e. circular electron machines of more than 100 GeV are not practical

- Electron moving towards observer with normalized velocity β emits wave with period T_e while observer sees a different period T_o

$$T_o = (1 - \mathbf{n} \cdot \boldsymbol{\beta}) T_e$$



- The wavelength becomes in the same way

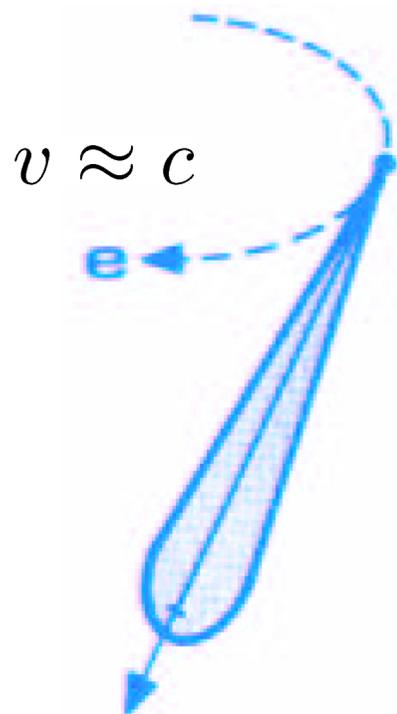
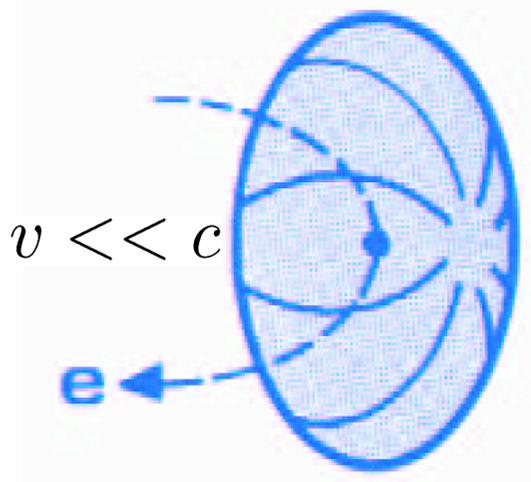
$$\lambda_o = (1 - \beta \cos \theta) \lambda_e$$

- Looking along the tangent of the trajectory $\theta = 0$ and by

using $1 - \beta = \frac{1 - \beta^2}{1 + \beta} \approx \frac{1}{2\gamma^2}$ the wavelength is $\lambda_o = \frac{1}{2\gamma^2} \lambda_e$

- The emitted wavelength is compressed by a large factor

- Taking into account electrons of a few GeV, (γ of a few 10^3) with wavelengths of a few cm, provide radiation of a nm (X-rays)



- For a non-relativistic source (or in the laboratory frame) radiation is axially symmetric, proportional to $\sin^2 \theta_e$ (Herz dipole)
- For relativistic source, the observed angle with respect to the emission angle is

$$\tan \theta_o = \frac{\sin \theta_e}{\gamma(\cos \theta_e - \beta)}$$

- For small angles $\theta_o = \frac{1}{\gamma} \theta_e$
- The radiation is emitted into a narrow cone, perpendicular to the electron trajectory

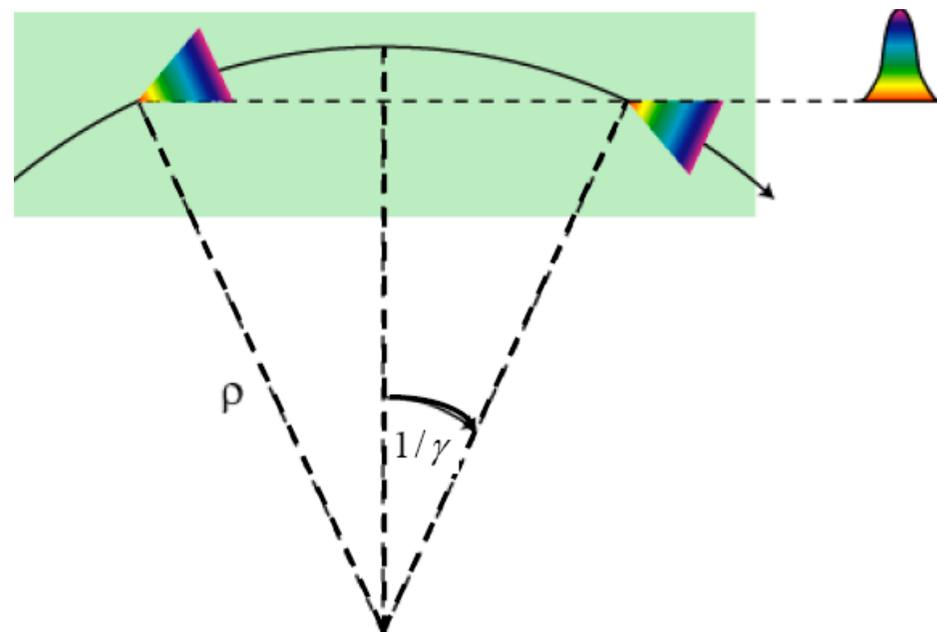
- Assume electrons moving in a ring of radius R
- Due to angle collimation, observer sees small fraction of electron trajectory $l = 2R/\gamma$
- The pulse length, defined as the time difference a photon and electron to cover this distance

$$\Delta t \approx \frac{l}{\beta c} - \frac{l}{c} = \frac{2R(1 - \beta)}{\gamma\beta c}$$

- Finally, the pulse length is

$$\Delta t \approx \frac{R}{c\gamma^3}$$

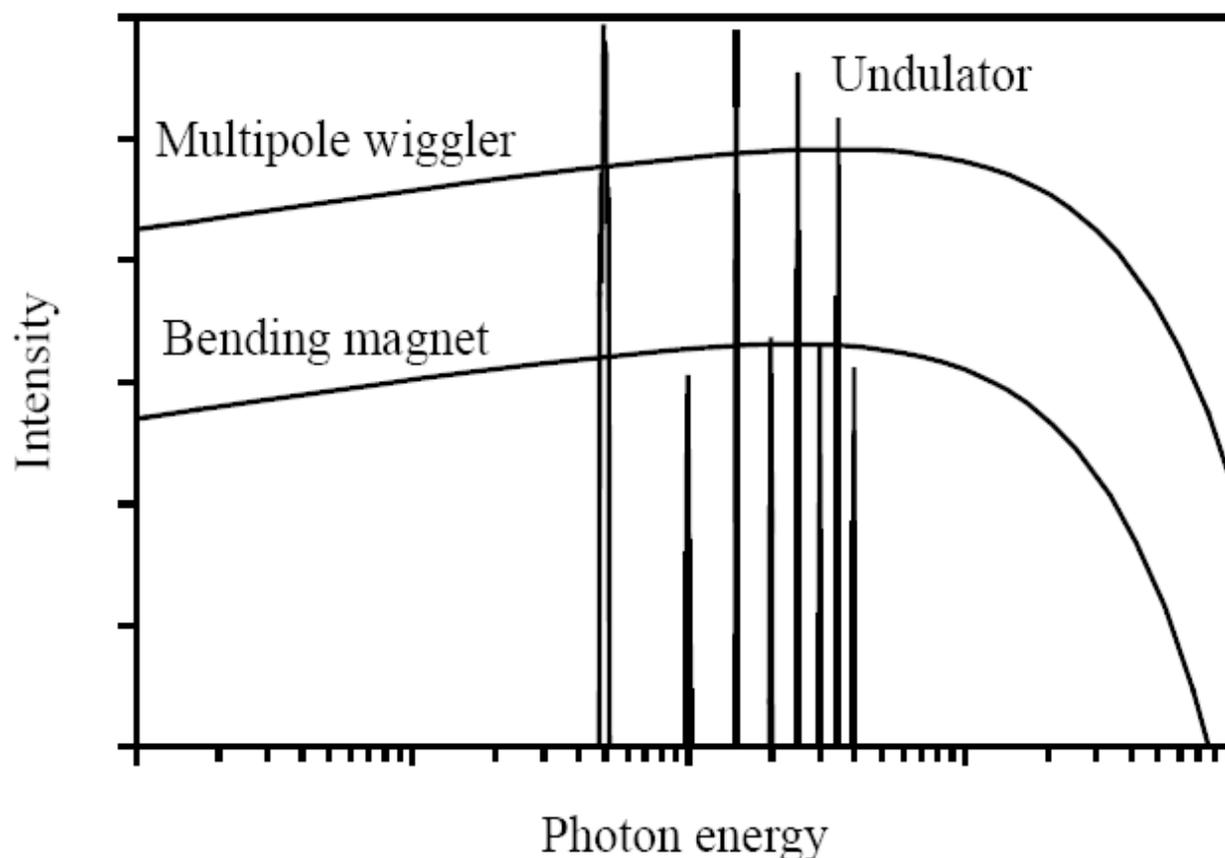
- This is a very short pulse (typically fraction of ns) and thus the radiation can be produced with a time structure

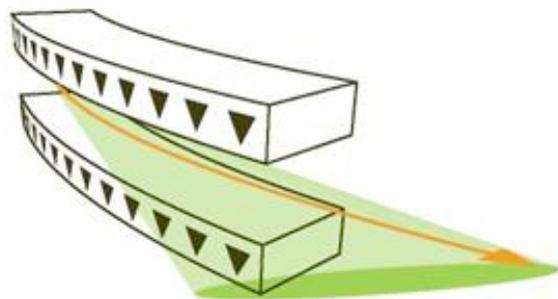


- The radiation comes in a series of flushes with a (critical or characteristic) frequency proportional to the revolution frequency

$$\omega_c \approx \gamma^3 \omega_0$$

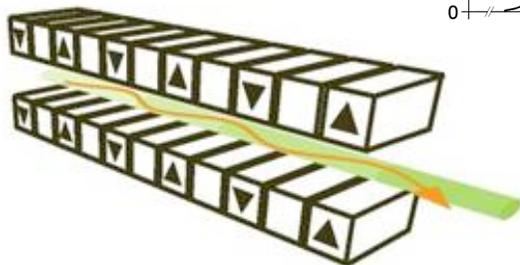
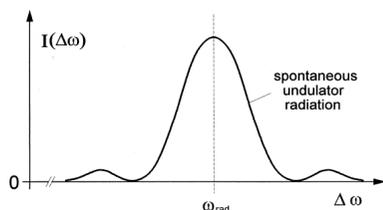
- This is almost a continuous spectrum, as the harmonics are so high that they overlap





Bending magnet (Sweeping searchlight)
At each deflection of the electron path a beam of radiation is produced.

Insertion devices



Undulator ($K \leq 1$)

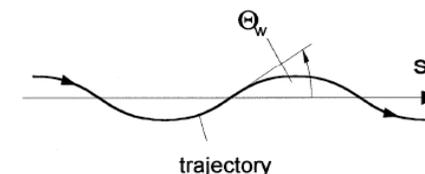
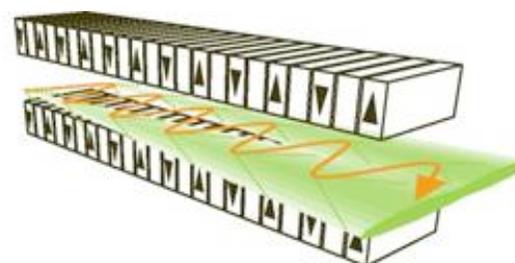
Produces a very narrow beam of coherent light

Undulator / wiggler parameter

$$K = \frac{\lambda_u e \tilde{B}}{2\pi m_e c}$$

Deflection angle

$$\Theta_W = \frac{K}{\gamma}$$



Wiggler ($K > 1$)

Beams emitted at each pole reinforce each other and appear as a broad beam of incoherent light.

$$\frac{dF_n}{d\lambda} = \int \frac{d\Phi_n}{d\theta_x d\theta_z} (\theta_x, \theta_z, \lambda_n) d\theta_x d\theta_z = \pi \alpha \frac{I}{e} N Q_n(K)$$

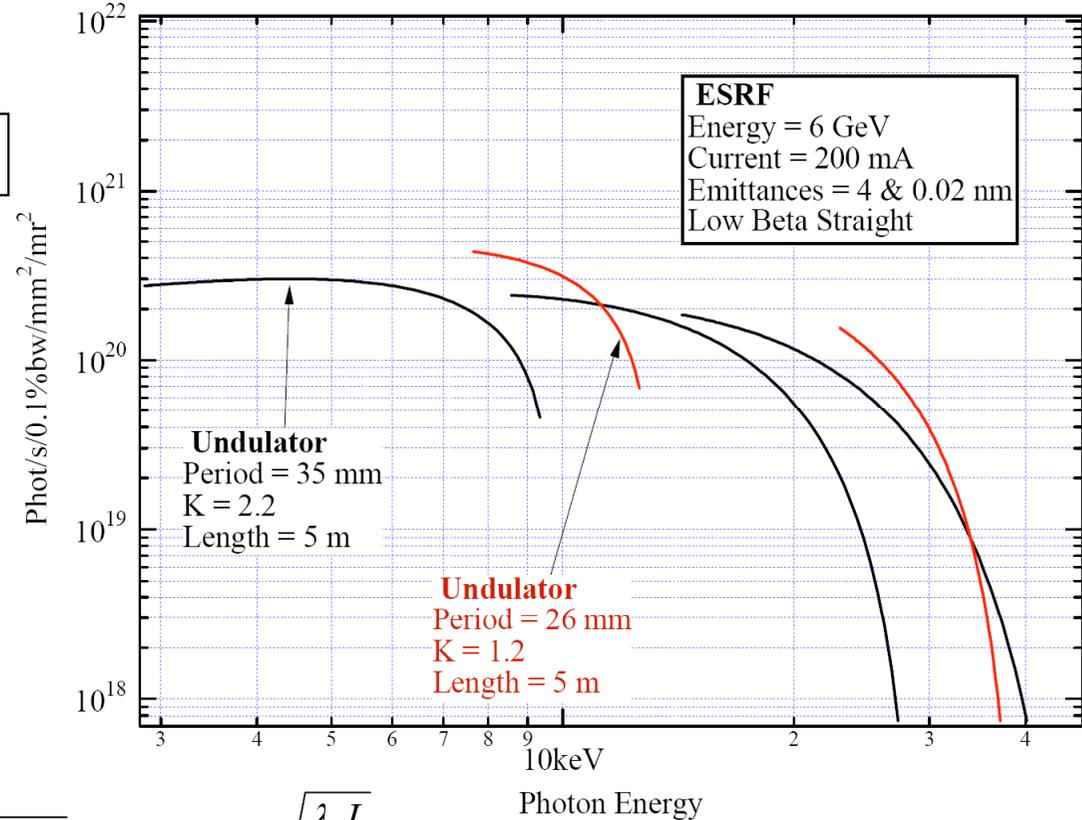
Maximum angular spectral flux

$$F_n [Ph/sec/0.1\%] = 1.43110^{14} N I[A] Q_n(K)$$

N : Number of Undulator Periods

I : Ring Current

n : Harmonic Number



Spectral brilliance and brightness

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_R^2}, \quad \sigma_R = \frac{\sqrt{\lambda_n L}}{2\pi}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \sigma'^2_R}, \quad \sigma'_R = \frac{\lambda_n}{2L}$$

Σ_x, Σ_z : Photon beam sizes

Σ'_x, Σ'_z : Photon beam divergences

